

SL-2 Math

Summer

Review

Welcome to SL-2!

Often times over the summer it is easy to forget some of the things you have learned. Here at Calverton we like to send work home to be completed throughout the summer in order to help students start their next year off strong.

Please complete the attached worksheets throughout the summer and avoid completing them all in the week before school starts. Please make sure you show all your work along the way.

Please read the two IA's, annotate by marking where you see or do not see the criteria, and fill out the grading sheet with reasons for each value.

The worksheet and the IA's will count as your first grades and needs to be completed for the first day of school.

There is a math help day for support in completing the required summer work.

Please email Mrs. Crissman acrissman@calvertonschool.org or Mr. Kerin wkerin@calvertonschool.org for any questions about the requirements.

Have a great summer!

1. Given that $z = \frac{10 \sin \alpha}{3x + y}$, where $\alpha = 30^\circ$, $x = 6$ and $y = 46$.

(a) Find the **exact** value of z .

[2]

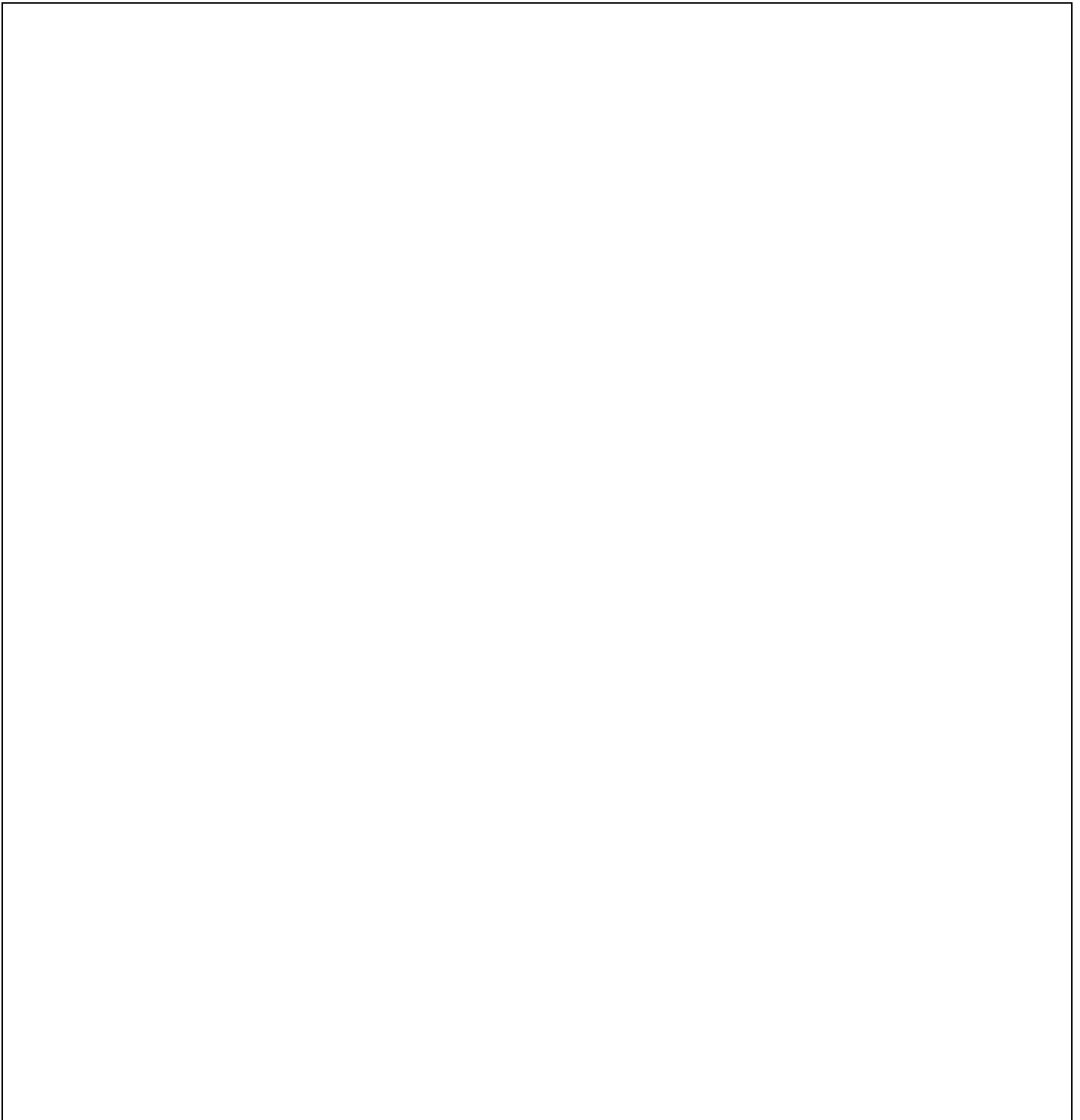
(b) Write your answer to part (a)

(i) correct to 2 decimal places;

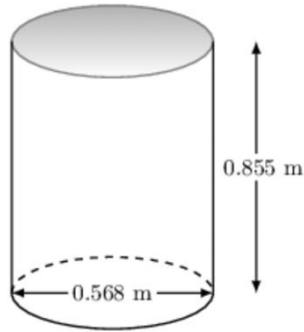
(ii) correct to 3 significant figures;

(iii) in the form $a \times 10^k$, where $1 \leq a < 10$ and $k \in \mathbb{Z}$.

[4]



2. A water storage tank has a cylindrical shape. The diameter of the base of the tank is 0.568 m. The height of the tank is 0.855 m. This is shown in the following diagram.



(a) Write down the radius, in m, of the base of the tank.

[1]

(b) Calculate the area of the base of the tank.

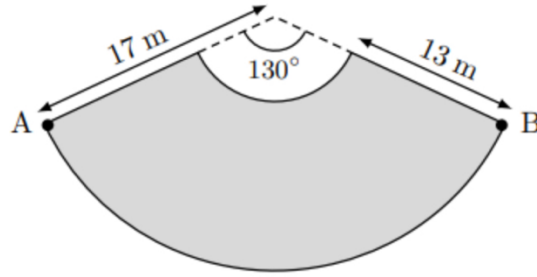
[2]

George is going to paint the curved surface and the base of the water storage tank.

(c) Calculate the area to be painted.

[3]

3. A rotating sprinkler waters part of a lawn, shown as the shaded area in the diagram below. The sprinkler covers a distance of 13 m, reaching a maximum distance of 17 m and covering an angle of 130° .

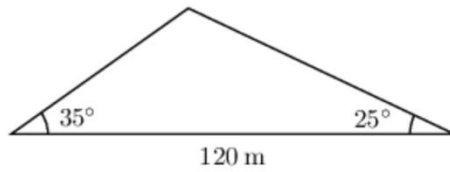


(a) Calculate the length of the outer arc the sprinkler can reach, the arc from point A to point B. [2]

(b) Find the area the sprinkler can cover. Give your answer to the nearest square metre. [3]

Blank area for student response.

4. A triangular plot of farming land is shown in the diagram below. The longest side is 120 m, with the two adjoining angles to this length being 35° and 25° .



- (a) Find the lengths of two other sides of this plot of land. [4]
- (b) Find the area of the plot of farming land. [2]

A large empty rectangular box provided for the student to show their work and calculations for parts (a) and (b).

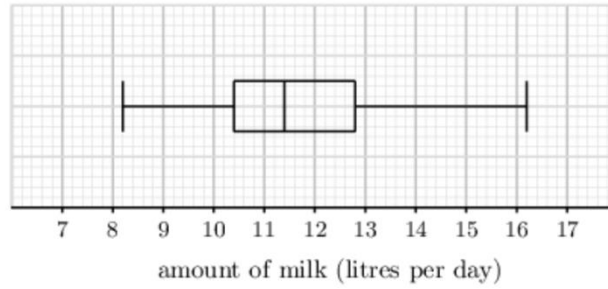
5. Aaron lives in a remote country town in outback Australia. He uses Amazon.com to purchase everyday supplies. Due to his remote location, it can take numerous days for deliveries to arrive. Aaron records the number of days for an order to arrive on his next ten orders, with the results as follows.

4, 5, 3, 2, 6, 11, 4, 4, 6, 7

For this data set, find the value of

- (a) the median; [2]
- (b) the mean; [2]
- (c) the standard deviation. [1]

6. Peter owns a dairy farm in New Plymouth, New Zealand, where hundreds of cows are bred for milk. In an effort of evaluating the cows productivity, he recorded the amount of milk that the cows produce over several days. The following box-and-whisker diagram represents the summary of the data.



- (a) Write down the median amount of milk that a dairy cow produces per day at his farm. [1]
- (b) Write down the lower and upper quartiles. [2]
- (c) Find the interquartile range. [1]

The amount of milk that these cows produce each day is known as being normally distributed.

- (d) Find the lowest amount of milk that a cow can produce and still not be considered an outlier. [2]

7. The following table shows the population of Philadelphia, correct to three significant figures, according to census data. The data is recorded at thirty-year intervals.

Year (x)	1830	1860	1890	1920	1950	1980	2010
Population (y)	85 000	566 000	1 050 000	1 820 000	2 070 000	1 690 000	1 530 000

Adam creates a linear model by finding the equation of the line passing through the points (1830, 85 000) and (1980, 1 690 000).

- (a) Calculate the gradient of the line passing through these two points. [2]
- (b) (i) Interpret the meaning of the gradient in the context of the question.
(ii) State appropriate units for the gradient. [2]
- (c) Find the equation of the line. Give your answer in the form $y = mx + c$. [2]
- (d) Use Adam's model to estimate the population of Philadelphia in the year 2000. [2]

Chuck obtains a model for the data by using linear regression.

- (e) (i) Find the equation of the regression line y on x .
(ii) Find the value of Pearson's product-moment correlation coefficient, r . [3]
- (f) Use Chuck's model to estimate the population of Philadelphia in the year 2000. [2]

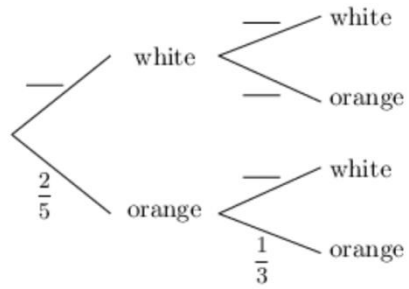
Chuck uses his model to predict the year when the population of Philadelphia will first reach 2 500 000.

- (g) State two reasons why Chuck's prediction may not be valid. [2]

8. A bag contains 6 white and 4 orange table tennis balls. Jack selects a ball at random from the bag and then, afterwards, John selects a ball at random from the bag.

(a) Complete the tree diagram.

[3]



(b) Find the probability that John chooses a white ball.

[3]

9. In the Monty Hall game show, there are three doors that hide either a goat or a car. Two of the doors hide a goat and the remaining door hides the car. In the game, each player is asked to choose one door at random. Suppose that there are 15 players playing the game in a particular month.

(a) Find the expected number of players who win a car.

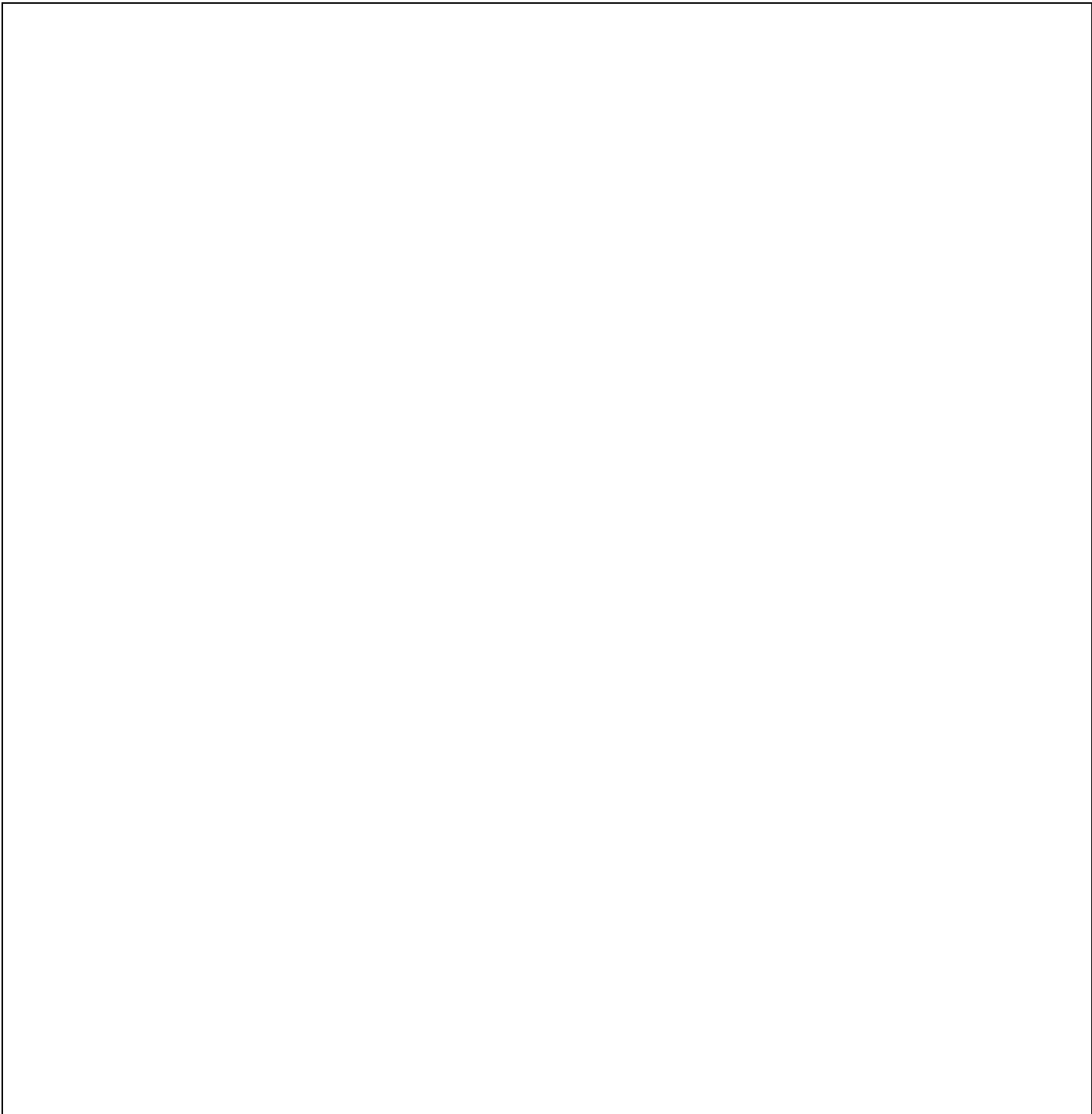
[2]

(b) Find the probability that

(i) exactly five players win a car;

(ii) more than five players win a car.

[4]



10. A pharmaceutical company has developed a new drug which they claim can lower blood sugar levels in diabetic patients.

Dr. Brown wants to test this claim. She did this by splitting a group of diabetes patients into two groups.

Group A, with 7 participants, were given the old drug and group B, with 8 participants, were given the new drug.

Dr. Brown found the blood sugar levels (mg/dL) for each patient after taking a course of the drug. The data is shown in the table below.

Group A (old drug)	17.0	15.9	13.0	13.9	14.1	13.0	14.2	
Group B (new drug)	12.3	14.9	17.0	12.2	13.9	13.4	11.5	11.5

She decided to conduct a two-sampled t -test at a 10% significance level to decide if the new drug is better at **reducing** blood sugar levels than the old drug.

The null hypothesis is $H_0 : \mu_A = \mu_B$, where μ_A is the mean blood sugar level of group A patients and μ_B is the mean blood sugar level of group B patients.

(a) State the alternate hypothesis for the test.

[1]

(b) Find the p -value for the test.

[2]

Dr. Brown concludes that there is sufficient evidence to reject the null hypothesis.

(c) State whether Dr. Brown is correct and give a reason for your answer.

[2]

SOLVE EACH EQUATION. CHECK YOUR ANSWER

1) $12 = 6(x + 3)$

8) $-4(x + 6) = 2x - 5$

2) $15(3 - 6x) + 24x = 45$

9) $5 - (x + 4) = -1 + 2x$

3) $12 = -4(14 - 2x)$

10) $2 - (x - 2) = 3(x + 6)$

4) $10(-2x - 8) = 20$

11) $7(2x + 1) = 4x + 2$

5) $4(2x + 6) = 20$

12) $\frac{x}{3} = 2(x + 5)$

6) $\frac{10x}{4} = 2x + 3$

13) $-\frac{x}{5} = 3x + 4$

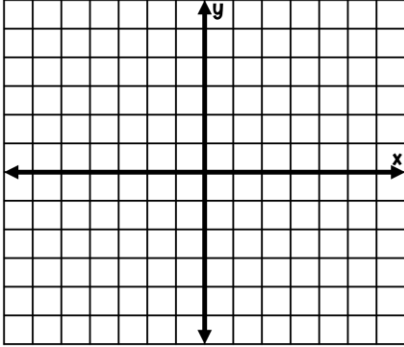
7) $\frac{x}{6} - 3 = 2 + x$

14) $\frac{7x}{3} = 12 + 2x$

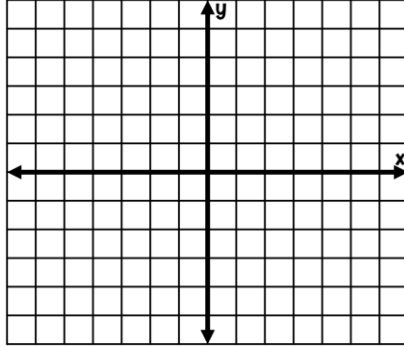
Graphing Linear Equations using SLOPE-INTERCEPT FORM

Graph each line.

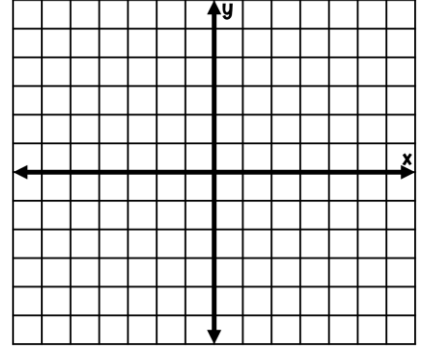
1 $y = \frac{1}{2}x + 3$



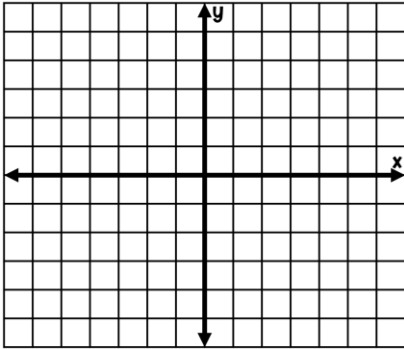
2 $y = \frac{3}{2}x - 4$



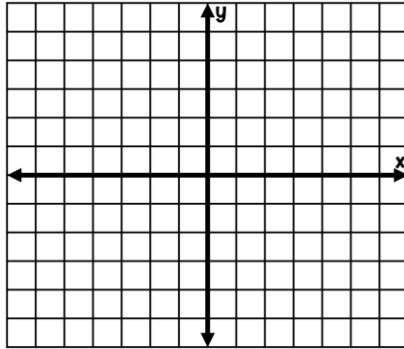
3 $y = \frac{4}{3}x - 5$



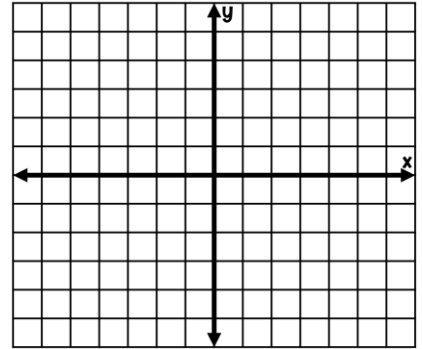
4 $y = 5x - 2$



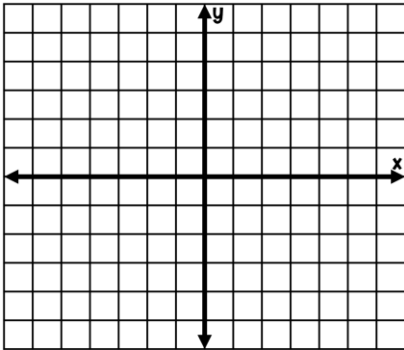
5 $y = 3x + 4$



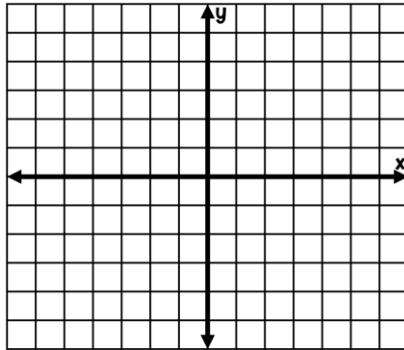
6 $y = -\frac{1}{3}x + 4$



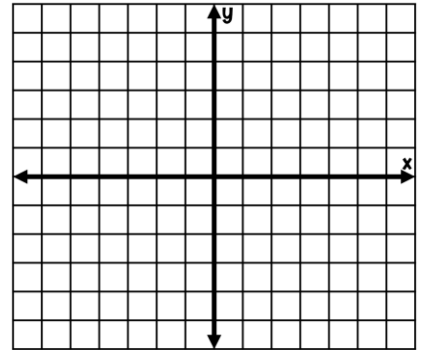
7 $y = 4x - 3$



8 $y = -\frac{1}{4}x$

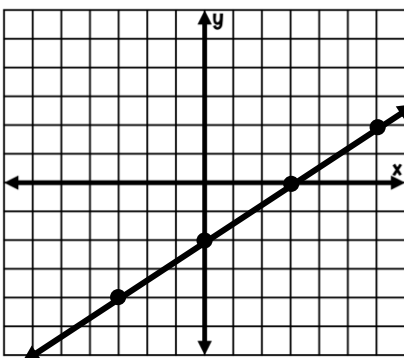


9 $y = x + 3$

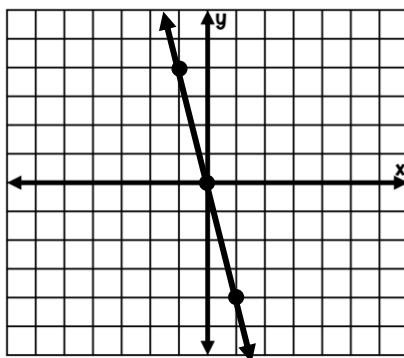


Write an equation in slope-intercept form for each line graphed below.

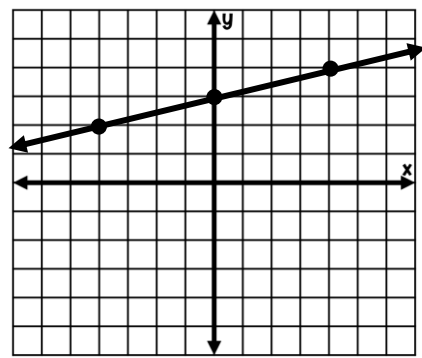
10 _____



11 _____



12 _____



Math Internal Assessment Scoring Criteria

Presentation	Mathematical Communication	Personal Engagement	Reflection	Use of Mathematics	Total
4 (20%)	4 (20%)	3 (15%)	3 (15%)	6 (30%)	20 (100%)

Presentation

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration has some coherence or some organization.
2	The exploration has some coherence and shows some organization.
3	The exploration is coherent and well organized.
4	The exploration is coherent, well organized, and concise

The “presentation” criterion assesses the organization and coherence of the exploration.

A **coherent** exploration is logically developed, easy to follow and meets its aim. This refers to the overall structure or framework, including introduction, body, conclusion and how well the different parts link to each other.

A **well-organized** exploration includes an introduction, describes the aim of the exploration and has a conclusion. Relevant graphs, tables and diagrams should accompany the work in the appropriate place and not be attached as appendices to the document. Appendices should be used to include information on large data sets, additional graphs, diagrams and tables.

A **concise** exploration does not show irrelevant or unnecessary repetitive calculations, graphs or descriptions.

The use of technology is not required but encouraged where appropriate. However, the use of analytic approaches rather than technological ones does not necessarily mean lack of conciseness, and should not be penalized. This does not mean that repetitive calculations are condoned.

Mathematical Communication

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	The exploration contains some relevant mathematical communication which is partially appropriate.

2	The exploration contains some relevant appropriate mathematical communication.
3	The mathematical communication is relevant, appropriate and is mostly consistent.
4	The mathematical communication is relevant, appropriate and consistent throughout.

The “mathematical communication” criterion assesses to what extent the student has:

- used appropriate mathematical language (**notation, symbols, terminology**). Calculator and computer notation is acceptable only if it is software generated. Otherwise it is expected that students use appropriate mathematical notation in their work
- defined **key terms** and variables, where required
- used **multiple forms of mathematical representation**, such as formulae, diagrams, tables, charts, graphs and models, where appropriate
- used a **deductive method** and set out proofs logically where appropriate

Examples of level 1 can include graphs not being labelled, consistent use of computer notation with no other forms of correct mathematical communication.

Level 4 can be achieved by using only one form of mathematical representation as long as this is appropriate to the topic being explored. For level 4, any *minor* errors that do not impair clear communication should not be penalized.

Personal Engagement

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of some personal engagement.
2	There is evidence of significant personal engagement.
3	There is evidence of outstanding personal engagement.

The “personal engagement” criterion assesses the extent to which the student engages with the topic by exploring the mathematics and making it their own. It is not a measure of effort.

Personal engagement may be recognized in different ways. These include thinking independently or creatively, presenting mathematical ideas in their own way, exploring the topic from different perspectives, making and testing predictions. Further (but not exhaustive) examples of personal engagement at different levels are given in the teacher support material (TSM).

There must be evidence of personal engagement demonstrated in the student’s work. It is not sufficient that a teacher comments that a student was highly engaged.

Textbook style explorations or reproduction of readily available mathematics without the candidate's own perspective are unlikely to achieve the higher levels.

Significant: The student demonstrates authentic personal engagement in the exploration on a few occasions and it is evident that these drive the exploration forward and help the reader to better understand the writer's intentions.

Outstanding: The student demonstrates authentic personal engagement in the exploration in numerous instances and they are of a high quality. It is evident that these drive the exploration forward in a creative way. It leaves the impression that the student has developed, through their approach, a complete understanding of the context of the exploration topic and the reader better understands the writer's intentions.

Reflection

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	There is evidence of limited reflection.
2	There is evidence of meaningful reflection.
3	There is substantial evidence of critical reflection.

The "reflection" criterion assesses how the student reviews, analyses and evaluates the exploration. Although reflection may be seen in the conclusion to the exploration, it may also be found throughout the exploration.

Simply describing results represents **limited reflection**. Further consideration is required to achieve the higher levels.

Some ways of showing **meaningful reflection** are: linking to the aims of the exploration, commenting on what they have learned, considering some limitation or comparing different mathematical approaches.

Critical reflection is reflection that is crucial, deciding or deeply insightful. It will often develop the exploration by addressing the mathematical results and their impact on the student's understanding of the topic. Some ways of showing critical reflection are: considering what next, discussing implications of results, discussing strengths and weaknesses of approaches, and considering different perspectives.

Substantial evidence means that the critical reflection is present throughout the exploration. If it appears at the end of the exploration it must be of high quality and demonstrate how it developed the exploration in order to achieve a level 3.

Use of Mathematics

Achievement level	Descriptor
0	The exploration does not reach the standard described by the descriptors below.
1	Some relevant mathematics is used. Limited understanding is demonstrated.
2	Some relevant mathematics is used. The mathematics explored is partially correct. Some knowledge and understanding is demonstrated.
3	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Some knowledge and understanding are demonstrated.
4	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct. Good knowledge and understanding are demonstrated
5	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is correct and demonstrates sophistication or rigour. Thorough knowledge and understanding are demonstrated.
6	Relevant mathematics commensurate with the level of the course is used. The mathematics explored is precise and demonstrates sophistication and rigour. Thorough knowledge and understanding are demonstrated.

The “Use of mathematics” HL criterion assesses to what extent students use **relevant** mathematics in the exploration.

Students are expected to produce work that is **commensurate with the level** of the course, which means it should not be completely based on mathematics listed in the prior learning. The mathematics explored should either be part of the syllabus, at a similar level or slightly beyond. However, mathematics of a level slightly beyond the syllabus is **not** required to achieve the highest levels.

A key word in the descriptor is **demonstrated**. The command term demonstrate means to make clear by reasoning or evidence, illustrating with examples or practical application. Obtaining the correct answer is not sufficient to demonstrate understanding (even some understanding) in order to achieve level 2 or higher.

For knowledge and understanding to be thorough it must be demonstrated throughout. Lines of reasoning must be shown to justify steps in the mathematical development of the exploration.

Relevant refers to mathematics that supports the development of the exploration towards the completion of its aim. Overly complicated mathematics where simple mathematics would suffice is not relevant. The mathematics can be regarded as **correct** even if there are occasional minor errors as long as they do not detract from the flow of the mathematics or lead to an unreasonable outcome. **Precise** mathematics is error-free and uses an appropriate level of accuracy at all times.

Sophistication: To be considered as sophisticated the mathematics used should be commensurate with the HL syllabus or, if contained in the SL syllabus, the mathematics has been used in a complex way that is beyond what could reasonably be expected of an SL student. Sophistication in mathematics may include understanding and using challenging mathematical concepts, looking at a problem from different perspectives and seeing underlying structures to link different areas of mathematics.

Rigour involves clarity of logic and language when making mathematical arguments and calculations. Mathematical claims relevant to the development of the exploration must be justified or proven.

Students are encouraged to use technology to obtain results where appropriate, but **understanding must be demonstrated** in order for the student to achieve level 1 or higher, for example merely substituting values into a formula does not necessarily demonstrate understanding of the results.

The mathematics only needs to be what is required to support the development of the exploration. This could be a few small elements of mathematics or even a single topic (or sub-topic) from the syllabus. It is better to do a few things well than a lot of things not so well. If the mathematics used is relevant to the topic being explored, commensurate with the level of the course and understood by the student, then it can achieve a high level in this criterion.

Subject: Mathematics, Applications and Interpretation

Title:

Student code: _____

Criteria	A	B	C	D	E	Total
Score						

Criteria	Score	Teacher comments <i>(use the rubric for terms, elements and criteria reached)</i>
A	x/4	
B	x/4	
C	x/3	
D	x/3	
E	x/6	

Conditions to note that may have affected this IA:

Is social media really that social?

A statistical analysis to measure how social media affects our personality and confidence.

Contents

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Introduction

I can relate to many teenagers today by admitting the fact that I am a heavy social media user. Over the past few weeks, I have read many articles that have been headlining how influential social media is towards affecting people's personality and confidence. From this I did not just become concerned about how I might be affected, but also how it may affect other teenagers or even adults. According to WhatIs.com, social media "is the collective of online communications channels dedicated to community-based input, interaction, content sharing and collaboration" (WhatIs, 2016) All sounds very social, however according to articles from USA Today, Science Daily, and Readers Digest, have all made comments about how we humans are becoming narcissists.

Science Daily expressed individuals with narcissist syndrome to think of "themselves as being exceptionally talented, remarkable and successful. [Meaning] they love to present themselves to other people and seek approval from them" (Gnambs, 2017) Reading this article was shocking, because I asked myself the question "am I really that desperate?" This lead me to compose an investigation on whether social media was affecting two key expressive traits, that being personality and confidence. Even though we may be living in 'generation me', where we could easily assume from that we are highly confident and mixed in personality types. I believe that world only exists on social media, because we have a screen to protect us from either the good or bad expression made by others. This protection must have some deliberate affect towards our real confidence or personality traits.

The observation made above has allowed this investigation to find out whether the amount of usage time spent on social media has an affect towards individual's belief of personality

and confidence. From this it will answer the question whether a high amount of social media usage does or does not affect personality in consideration to both age and gender.

Plan of investigation

To test whether there is a relationship between hours spent on social media and gender, age and believed personality type (extrovert or introvert). The survey will be designed on ‘Google Forms’, and posted on Facebook to receive a significant amount of data that can be analysed through three mathematical processes. Posting the survey on Facebook will allow for an inflow of relatively precise data because the information received is correctly aimed to ask the most appropriate audience (social media users).

The data will then be cleaned from any inappropriate and inconclusive submissions. For example, after the one week of surveying, the investigation was left with seventy-seven of the one hundred and twenty-five submissions. Forty submissions were male, where majority aged between 15 and 20. Thirty-seven submissions were female, where again majority aged between 15 and 20. Pleased with the data received, as there was very minimal bias entries associated with age, and gender.

The survey will be designed on Google Forms because they can create questions that aim for narrow-minded answers. The survey involves three stages, hence the first stage includes general questions regarding age, gender, most used social media app and how many hours are spent on social media a day. The second stage is a personality test, referenced from a well-regarded psychiatric group called Psych Central (Grohol J. M., 2016). In the test, made by Dr. John Grohol, designed it so the sampler will receive questions asking how strongly they feel towards a range of personality qualities. For example, “I see

myself as critical, quarrelsome.” The sampler can then score themselves between 1 to 10. 1 indicates a strong disagreement towards the statement, whereas 10 strongly agrees with it. The third stage was a confidence test that asked a set of hypothetical situations, and then gave the sampler options on how they would respond to the situation. The individual with a large amount of yes’s, shows high confident levels, whereas an individual with many no’s demonstrates them to have low confidence levels.

Once the data is collected, three mathematical processes are used in search for the answer to the proposed question. First process is a box and whisker graph to compare gender and hours spent on social media. The second process is a chi-squared test for testing independence between hours spent on social media and self-indication of personality type (introvert or extrovert). Lastly the third process will use Pearson’s Product Momentum Correlation Coefficient to find the strength of the correlation between age and hours spent on social media. After processing all mathematical methods, we can develop a conclusion whether large hours spent on social media has correlation with either age, gender or personality type.

Investigation

Box and Whisker Plot for Gender and Hours Spent on Social Media:

To compare the whether there is a relationship between hours spent on social media and gender. The mathematical model Box and Whisker Plot, will allow for comparison between the two variables. From the data, the minimum value, lower quartile, median, upper quartile, and maximum value are present. The values are spread along the continuous scale between two single axes.

Mean Female time spent on social media per day score

$$X = \frac{\sum fx}{n}$$

$$X = \frac{140.2}{37}$$

$$X = 3.47$$

On average women spend 3.47 hours a day on social media.

Mean Male time spent on social media per day score

$$X = \frac{\sum fx}{n}$$

$$X = \frac{127}{40}$$

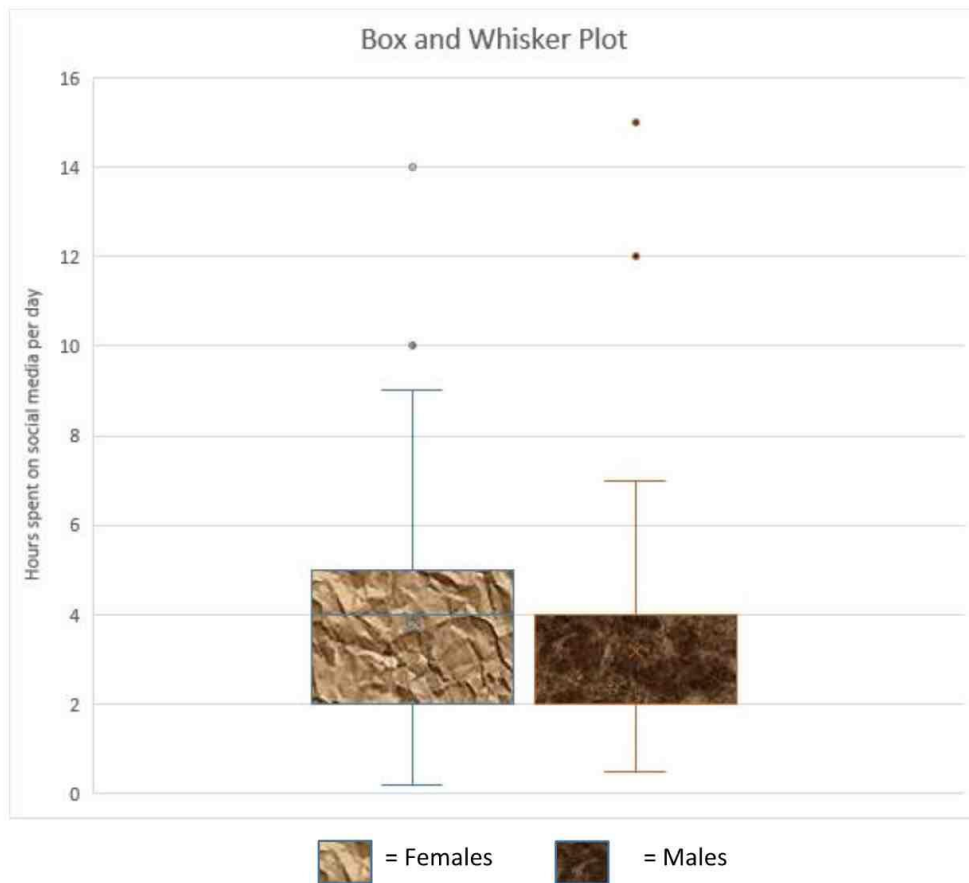
$$X = 3.175$$

Whereas on average men spend 3.20 hours per day on social media.

The raw data collected produced the following box and whisker graph:

Females	
Sum	140.2
Mean	3.47
Minimum	0.2
Q1	2
Median	4
Q3	5
Maximum	14
Range	13.8
IQR	3

Males	
Sum	127
Mean	3.175
Minimum	0.5
Q1	2
Median	2
Q3	4
Maximum	15
Range	14.5
IQR	2



The box and whisker plot above shows that men and women express two different medians and mean scores. Hence, women appear to show a much larger median score than men. For example, males have a median score of 2, whereas women double this and indicate a score of 4. This means that over 50% of women spend more than 4 hours a day on social media because the median is located more towards the higher spectrum of the box. Whereas less than 50% of men spend up to 2 hours a day on social media because the median is seen to be at the lower end of the spectrum. This demonstrates that women on average spend a greater time on social media because they indicate a much larger median figure.

The Box and Whisker plot above also demonstrates the males 'range' appears to be much larger than the women's. This means, a range of 14.5 (males) and 13.8 (females) directs this mathematical method to claim that the data received from males was much spread compared to women. In other words, women share a high level of agreement with each other, therefore supporting the statement that women collectively share a large number of hours spent on social media compared to men. However, an inner quartile range of 2 hours (males) and 3 hours (females) showed that much of the data was clumped, hence strengthen the argument that women tend to spend more time on social media than men.

With women demonstrating a mean of 3.47, and men indicating a mean of 3.175. Expresses to us that on average women spend 0.295 more hours than men on social. Therefore, the data suggests there is a significant difference between social media usage time for men and women. Where we can conclude that there is a relationship between hours spent on social media and gender of users.

Chi-squared Test for Testing the independence of hours spent on social and self-believed identification of personality type for social media users:

To compare hours spent on social media and personality type, a chi-squared (X^2) can be used to compare two discrete values. A group of 77 males and females have been split into self-believed personality types – extrovert or introvert. An introvert is a person who is characterised by his or her thoughts, whereas an extrovert is primarily concerned with the physical and social environment (dictionary.com, 2017). Class 1-5 are individuals who strongly believe they are introverts, therefore we label them as introverts. Whereas class 6-10 strongly believe they are extroverts, hence will be labelled as extroverts. For this investigation, we are interested in finding whether the amount of time (hours) spent on our phones affects our personality type (introvert or extrovert).

The survey conducted was a self-conducted survey, meaning people could have answered the hypothetical introvert or extrovert questions at their own will. For example, “I see myself open to new experiences, complex.” (Grohol J. M., 2016) From this, if an individual disagreed or agreed to this, we can compare that self-identification to how many hours they spend on social media.

Cleaning the data was necessary for this process because we did receive some responses where they spent more than 24 hours on social media per day. This is clearly unjustifiable and therefore will not be incorporated as they are insignificant. From previous answers, some participants had to be ruled out of the investigation because they were deemed inappropriate and non-conclusive.

Hypothesis:

Null Hypothesis (H_0) Time spent on social media is independent to personality type.

Alternate Hypothesis (H_1) – Time spent on social media is not independent to personality type.

Observed Frequencies:

	Low Usage (0-5)	Medium Usage (5.1 to 10)	High Usage (10.1 - 15)	Total
Introvert	19.00	14.00	1.00	34.00
Extrovert	40.00	2.00	1.00	43.00
Total	59.00	16.00	2.00	77.00

The table above represents how the number of hours on social media affects personality type. For example, there is 19 individuals who self-identified themselves as introverted, and had a low social media usage time. But, this was overruled by the staggering 40 individuals who self-identified themselves as extroverts and had a low social media usage time.

Expected Frequencies:

To calculate expected frequency $\frac{\text{Row total} \times \text{column total}}{\text{grand total}}$

	Low Usage (0-5)	Medium Usage (5.1 to 10)	High Usage (10.1 - 15)	Total
Introvert	26.05	7.06	0.88	34.00
Extrovert	32.95	8.94	1.12	43.00
Total	59.00	16.00	2.00	77.00

In attempt to search for the Chi-Squared value, this investigation will use the Yate’s continuity correction. The formula below will determine the value of Chi-Squared.

$$X^2 = \sum_{i=1}^n \frac{(O_i - E_i - 0.5)^2}{E_i}$$

(How2stats, 2011)

O_i	E_i	$O_i - E_i$	$ O_i - E_i $	$ O_i - E_i - 0.5$	$(O_i - E_i - 0.5)^2$	$\frac{(O_i - E_i - 0.5)^2}{E_i}$
19	26.05	-7.05	7.05	6.55	42.9025	1.646928983

40	32.95	7.05	7.05	6.55	42.9025	1.302048558
14	7.06	6.94	6.94	6.44	41.4736	5.874447592
2	8.94	-6.94	6.94	6.44	41.4736	4.639105145
1	0.88	0.12	0.12	-0.38	0.1444	0.164090909
1	1.12	-0.12	0.12	-0.38	0.1444	0.128928571

$$\sum_{i=1}^n \frac{(O_i - E_i - 0.5)^2}{E_i} = 13.756$$

$$X^2_{calc} = 13.756$$

Chi-squared Critical Value

Level of Significance

Degrees of Freedom	df	0.05	0.025	0.01	0.005
	1	6.314	12.706	31.821	63.653
	2	2.920	4.303	6.965	9.925
	3	2.353	3.18	4.541	5.841
	4	2.132	2.776	3.747	4.604
	5	2.015	2.571	3.365	4.032
	6	1.943	2.447	3.143	3.707
	7	1.895	2.365	2.998	3.499
	8	1.860	2.306	2.896	3.355
	9	1.833	2.262	2.821	3.250

(tutorvista.com, 2017)

When testing at a 2.5% level of significance and with 2 degrees of freedom, the critical value is **4.303**.

$$\text{Degrees of freedom } (v) = (\# \text{ of rows } - 1) \times (\# \text{ of columns } - 1)$$

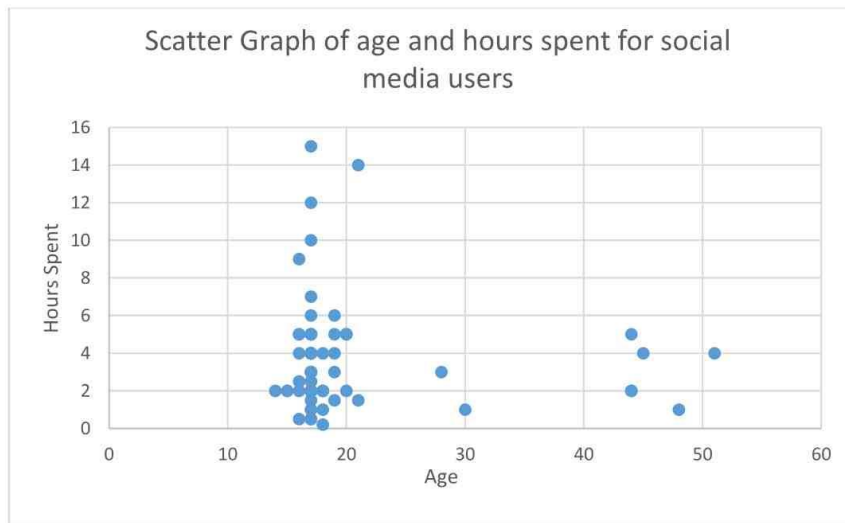
$$V = (2-1) \times (3-1)$$

$$V = 2$$

As the X^2_{calc} value of 13.756 is more than the X^2_{crit} of 4.303, we reject the null hypothesis that individuals who spend a low usage of time on social media is independent to personality type. From what is observed we can determine that individuals who spend a low usage of time on social media does not affect their personality. This possibly supports our

original statement that high concentrates of social media does affect personality because there was no effect shown for low usage social media users.

Pearson’s Product Momentum Correlation Coefficient for Age and Hours Spent on Social Media for Users:



Formula: $r = \frac{S_{xy}}{S_x S_y}$

Data 1

Data 2

Data 3

Age	Hours	xy	Age (x)	$x_i - \mu$	$(x_i - \mu)^2$	Hours (y)	$x_i - \mu$	$(x_i - \mu)^2$
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
17	4	68	17	-2.81	7.8961	4	0.53	0.2809
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
30	1	30	30	10.19	103.8361	1	-2.47	6.1009
14	2	28	14	-5.81	33.7561	2	-1.47	2.1609
15	2	30	15	-4.81	23.1361	2	-1.47	2.1609
16	2	32	16	-3.81	14.5161	2	-1.47	2.1609
17	6	102	17	-2.81	7.8961	6	2.53	6.4009
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
44	2	88	44	24.19	585.1561	2	-1.47	2.1609
18	2	36	18	-1.81	3.2761	2	-1.47	2.1609
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609

15	2	30	15	-4.81	23.1361	2	-1.47	2.1609
17	5	85	17	-2.81	7.8961	5	1.53	2.3409
17	7	119	17	-2.81	7.8961	7	3.53	12.4609
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
18	2	36	18	-1.81	3.2761	2	-1.47	2.1609
18	2	36	18	-1.81	3.2761	2	-1.47	2.1609
16	2.5	40	16	-3.81	14.5161	2.5	-0.97	0.9409
17	2.5	42.5	17	-2.81	7.8961	2.5	-0.97	0.9409
17	3	51	17	-2.81	7.8961	3	-0.47	0.2209
19	4	76	19	-0.81	0.6561	4	0.53	0.2809
19	1.5	28.5	19	-0.81	0.6561	1.5	-1.97	3.8809
44	5	220	44	24.19	585.1561	5	1.53	2.3409
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
17	4	68	17	-2.81	7.8961	4	0.53	0.2809
17	0.5	8.5	17	-2.81	7.8961	0.5	-2.97	8.8209
16	0.5	8	16	-3.81	14.5161	0.5	-2.97	8.8209
17	0.5	8.5	17	-2.81	7.8961	0.5	-2.97	8.8209
20	5	100	20	0.19	0.0361	5	1.53	2.3409
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
17	12	204	17	-2.81	7.8961	12	8.53	72.7609
17	15	255	17	-2.81	7.8961	15	11.53	132.9409
20	2	40	20	0.19	0.0361	2	-1.47	2.1609
17	4	68	17	-2.81	7.8961	4	0.53	0.2809
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
17	3	51	17	-2.81	7.8961	3	-0.47	0.2209
28	3	84	28	8.19	67.0761	3	-0.47	0.2209
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
20	5	100	20	0.19	0.0361	5	1.53	2.3409
17	4	68	17	-2.81	7.8961	4	0.53	0.2809
17	1	17	17	-2.81	7.8961	1	-2.47	6.1009
16	9	144	16	-3.81	14.5161	9	5.53	30.5809
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
17	10	170	17	-2.81	7.8961	10	6.53	42.6409
16	5	80	16	-3.81	14.5161	5	1.53	2.3409
18	1	18	18	-1.81	3.2761	1	-2.47	6.1009
51	4	204	51	31.19	972.8161	4	0.53	0.2809
17	1.5	25.5	17	-2.81	7.8961	1.5	-1.97	3.8809
48	1	48	48	28.19	794.6761	1	-2.47	6.1009
17	4	68	17	-2.81	7.8961	4	0.53	0.2809
18	4	72	18	-1.81	3.2761	4	0.53	0.2809
19	3	57	19	-0.81	0.6561	3	-0.47	0.2209

16	5	80	16	-3.81	14.5161	5	1.53	2.3409
17	3	51	17	-2.81	7.8961	3	-0.47	0.2209
17	5	85	17	-2.81	7.8961	5	1.53	2.3409
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
17	4	68	17	-2.81	7.8961	4	0.53	0.2809
18	0.2	3.6	18	-1.81	3.2761	0.2	-3.27	10.6929
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
21	1.5	31.5	21	1.19	1.4161	1.5	-1.97	3.8809
19	5	95	19	-0.81	0.6561	5	1.53	2.3409
17	5	85	17	-2.81	7.8961	5	1.53	2.3409
17	5	85	17	-2.81	7.8961	5	1.53	2.3409
17	4	68	17	-2.81	7.8961	4	0.53	0.2809
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
17	4	68	17	-2.81	7.8961	4	0.53	0.2809
45	4	180	45	25.19	634.5361	4	0.53	0.2809
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
44	2	88	44	24.19	585.1561	2	-1.47	2.1609
17	3	51	17	-2.81	7.8961	3	-0.47	0.2209
17	1	17	17	-2.81	7.8961	1	-2.47	6.1009
16	4	64	16	-3.81	14.5161	4	0.53	0.2809
21	14	294	21	1.19	1.4161	14	10.53	110.8809
19	6	114	19	-0.81	0.6561	6	2.53	6.4009
17	2	34	17	-2.81	7.8961	2	-1.47	2.1609
$\bar{x} =$ 19.81								
$\bar{y} = 3.47$								
$\overline{xy} =$ 67.75								

Step 1

Data used to calculate S_w :

Calculate $s_{xy} = \overline{xy} - (\bar{x} \times \bar{y})$

$S_{xy} = 67.75 - (19.81 \times 3.47)$

Step 2

Data used to calculate standard deviation of x :

Calculate $\sigma = \sqrt{\frac{\sum(xi - \mu)^2}{n}}$

$\sum(xi - \mu)^2 = 4875.46$

$$S_{xy} = -.9907$$

$$\sqrt{\frac{\sum(xi-\mu)^2}{n}} = \sqrt{\frac{4875.46}{77}}$$

$$\sqrt{\frac{\sum(xi-\mu)^2}{n}} = 63.31$$

Step 3

Data used to calculate standard deviation of y:

$$\text{Calculate } \sigma = \sqrt{\frac{\sum(xi-\mu)^2}{n}}$$

$$\sum(xi - \mu)^2 = 580.8213$$

$$\sqrt{\frac{\sum(xi-\mu)^2}{n}} = \sqrt{\frac{580.8213}{77}}$$

$$\sqrt{\frac{\sum(xi-\mu)^2}{n}} = 7.54$$

$$\text{Formula: } r = \left(\frac{S_{xy}}{S_x S_y} \right)$$

$$r = \frac{-.9907}{(63.31 \times 7.54)}$$

$$r = -0.0482$$

The value (r) is then squared to find the r^2 value from the following calculations. The r^2 value will be between 0 and 1, showing the strength of the correlation that exists between hours spent on social media and age.

$$r^2 = (-0.0482)^2$$

$$r^2 = 0.02323$$

With reference to the table below, we can determine the strength of the correlation.

Value	Strength
$r^2 = 0$	No Correlation
$0 < r^2 < 0.25$	Very Weak
$0.25 \leq r^2 < 0.5$	Weak
$0.5 \leq r^2 < 0.75$	Moderate
$0.75 \leq r^2 < 1$	Strong
$r^2 = 1$	Perfect Correlation

A r^2 value of 0.02323 demonstrates that the correlation between hours spent on social media and age is very weak. A very weak correlation shows age does not act as a determinate in how many hours an individual will spend on social media per day. (Lucas, 2012)

Conclusion

To conclude, the three mathematical processes carried out in this investigation did produce some evaluative points from the answers received. The evidence seemed to suggest that between the amount hours spent on social media, and gender is well as self-indicated personality type of users are independent to each other. Unfortunately, due to an insignificant amount of evidence some may conclude that the relationship between hours spent on social media and age is not independent.

However, the box and whisker graph did present some outstanding results showing how women on average tend to spend more time on social media than men. This was indicated through women spending a mean of 3.47 hours per day, whereas men spent a mean of 3.175 hours per day. Even though it was only a .295 of an hour difference, this can still exclaim how women in the data showed a greater usage time. Furthermore, this statement was proved by women demonstrating a smaller range of different hours spent on social media. For example, women had a range of 13.8 hours, whereas men were 14.5 hours. A smaller range concludes that the data from women was more clumped, therefore the mean given was stronger.

In the chi-squared test, outstanding results forced the investigation to reject the null hypothesis that individuals who spend a low usage of time on social media is independent to personality type. Reason being because the critical value of 4.303 was less than the calculated value of 13.756. The significant difference in values caused a rejection of the null hypothesis, appearing that people with low social media usage do not encounter any effects towards personality. Both mathematical methods mentioned above, answers' the original

question of whether a high amount of social media usage does or does not affect personality in consideration to both age and gender. It certainly does in terms of gender and self-indicated personality. Gender because women appeared to show a greater average number of hours spent on social media compared to men. Self-indication because individuals who spent less usage time on social media showed very consistent and strong personality qualities in the questionnaire. However, it can be concluded that there is no definite relationship between hours spent on social media and age due to the lack of supporting evidence. Therefore only 2 out of the 3 chosen methods answer our question, therefore we cannot fully determine whether a relationship exists between these two variables.

Validity

This report has many different limitations that could have deputed the overall outcome of the investigation.

Firstly, the data could have been strengthened if it was more balanced across age groups especially, and a greater quantity. E.g. For Pearson's Product Momentum Correlation Coefficient did produce a fair answer of how there was a very weak correlation between age and hours spent on social media. But this could have been overruled by the disproportion amount of age samples that were 17. In future, instead of posting the questionnaire on my own Facebook. Post it on others who vary in age, that way this would eradicate the significant disproportionate number of teenagers.

Secondly, the organisation of cleaning the data received was not professional. For example, in the box and whisker plot many of the outliers used for that mathematical process, was not taken out for other mathematical process as well. Therefore, this would have skewed

the overall investigation because the other two methods contained uncleaned data, as well as extra data. In future, this can be improved by first cleaning all data on one excel sheet that will work for all mathematical processes.

Lastly, would be to reword the original observation and make sense of 'self-indicated personality belief'. This investigation intended to show how the different concentrations of time spent on social media can cause negative or poor personality traits. The introduction could have done with a paragraph specifying why and how poor personality traits are generated through high concentrates of social media. In future, this could have improved through a better and more focussed plan.

17	5	1	0.5	Facebook	7	3	5	2	8	2	8	4	4	2	No	No	Yes	Depend: Sometim	Yes	Yes	Depends how big he/she was?
17	5	4	5	Facebook	8	4	7	3	7	1	8	7	4	4	Yes	No	Yes	Depend: Sometim	Yes	Yes	No
18	6	3	2	Instagrai	5	8	9	3	10	6	7	2	8	4	Yes	No	Yes	Sometim	Yes	Yes	Depends how big he/she was?
17	5	2	12	faceboo	3	6	6	7	5	6	4	3	7	6	Yes	a little	No	Depend: Sometim	Tiny/Am	Perhaps	No
21	6	10	15	instagrai	10	10	10	10	10	10	10	10	10	10	Yes	No	Yes	No	Yes	Yes	Yes
19	10	7	4	Snaphoi	8	6	7	2	10	2	3	5	7	4	Yes	No	Yes	No	Yes	Tiny/Am	Yes
17	6	4	2	Instagrai	6	4	8	8	8	5	8	3	6	5	Yes	No	Sometim	Depend: No	Yes	Yes	Depends how big he/she was?
17	6	4	2	Snaphoi	6	6	10	7	10	10	8	1	8	5	Yes	a little	Yes	Depend: Sometim	Yes	Yes	Depends how big he/she was?
17	7	4	2	Faceboo	5	8	5	2	6	5	1	6	8	4	Yes	a little	Yes	Depend: Yes	Yes	Yes	Depends how big he/she was?
17	7	5	4	Instagrai	8	8	7	7	10	8	9	4	5	2	Yes	No	Yes	Depend: Sometim	Tiny/Am	Perhaps	Yes
17	5	5	3	instagrai	3	2	7	3	8	8	9	7	6	2	Yes	No	Yes	No	No	Tiny/Am	Perhaps
45	9	1	1	Faceboo	9	1	8	2	9	2	9	3	8	4	Yes	a little	Yes	Yes	Yes	Tiny/Am	Yes
17	10	5	4	Faceboo	10	9	9	7	10	2	9	4	8	5	Yes	No	Sometim	Depend: Sometim	Yes	Perhaps	Depends how big he/she was?
44	9	4	2	wechat	10	6	7	3	9	4	9	3	8	3	No	a little	Sometim	No	Sometim	Tiny/Am	Perhaps
17	7	5	3	Faceboo	3	4	5	7	7	5	8	6	5	7	Yes	a little	Yes	Yes	No	Yes	Depends how big he/she was?
17	7	5	3	Faceboo	3	4	5	7	7	5	8	6	5	7	Yes	a little	Yes	Yes	No	Yes	Depends how big he/she was?
16	10	8	14	Faceboo	1	10	10	10	10	1	1	10	3	1	Yes	No	Yes	No	Yes	Yes	Yes
21	10	5	6	Instagrai	9	5	9	8	9	2	8	3	8	5	No	No	Yes	Depend: Sometim	Yes	Yes	No
19	4	3	2	Instagrai	7	10	10	1	1	6	6	8	10	3	Yes	No	Yes	No	Yes	Yes	Yes
17	10	2	2	Faceboo	10	3	8	3	9	1	9	8	9	9	Yes	No	Yes	Depend: No	Tiny/Am	Perhaps	Depends how big he/she was?

Questionnaire

Is Social media really that social?

Maths IA

Are you Male or Female?

- Male
- Female

How old are you?

Short answer text
.....

On a scale from 1 to 10, how popular do you think you are? 1 being 1 friend and 10 being over 20.

1	2	3	4	5	6	7	8	9	10	
Not Very Popular	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	<input type="radio"/>	Very Popular

How many social media accounts do you have?

Short answer text
.....

How many hours a day are you on social media?

Short answer text
.....

What social media app or site do you visit most?

Short answer text
.....

Section 2 of 3

Personality Test

Maths IA

1. I see myself as extroverted, enthusiastic.

1 2 3 4 5 6 7 8 9 10

Strongly Disagree Strongly Agree

I see myself as critical, quarrelsome.

1 2 3 4 5 6 7 8 9 10

Strongly Disagree Strongly Agree

I see myself as dependable, self-disciplined.

1 2 3 4 5 6 7 8 9 10

Strongly Disagree Strongly Agree

I see myself as anxious, easily upset.

1 2 3 4 5 6 7 8 9 10

Strongly Disagree Strongly Agree

I see myself as open to new experiences, complex.

1 2 3 4 5 6 7 8 9 10

Strongly Disagree Strongly Agree

I see myself as reserved, quiet.

Section 3 of 3



Confidence Test

Description (optional)

Would you consider to appear on a television show?

- Yes
- No

Would giving a long speech at your best friend's wedding completely embarrass you?

- Yes
- No
- a little

Have you ever disagreed with your boss or teacher at work?

- Yes
- No
- Sometimes

Does being naked in front of your friends bother you?

- Yes
- No
- Depends who

Are you ever ruthless?

- Yes
- No

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Subject: Mathematics, Applications and Interpretation

Title:

Student code: _____

Criteria	A	B	C	D	E	Total
Score						

Criteria	Score	Teacher comments <i>(use the rubric for terms, elements and criteria reached)</i>
A	x/4	
B	x/4	
C	x/3	
D	x/3	
E	x/6	

Conditions to note that may have affected this IA:

Modeling the Internal Temperature of Cookies Over Time

Introduction:

From a young age, I've been an enthusiast for everything chocolate—chocolate ice cream, brownies, and of course, the classic chocolate chip cookies. Rather than purchasing the chocolate chip cookies at the grocery store, I prefer to bake them myself. The cookies are not only fresher and more delicious, but I value the time I spend baking as I find it to be a very relaxing and gratifying activity. However, despite putting my cookies in the oven for the amount of time the recipe suggests, they usually turn out burnt on the bottom, causing too many baking sessions that started with eagerness to end in frustration. Therefore, I have resolved to fix the problem of burnt cookies by using math to discover the optimal amount of time chocolate chip cookies should be baked for to ensure that they're not undercooked or burnt, but perfect.

While researching how to ensure my cookies don't burn, I wondered if there was another quantifiable way to signify when a cookie was done baking, aside from trusting the suggested time on the recipe. I thought of temperature since it's easy to measure and relevant to baking; this sparked the idea of measuring the internal temperature of the cookies until it reaches a specific temperature whereby all raw ingredients in the dough are cooked and safe to eat—but what temperature is that? Because almost all cookie dough consists of raw eggs, I have to be certain that the raw eggs are baked to a safe temperature since raw eggs can be harmful to the body due to bacteria within them that can cause food poisoning. According to Health Canada¹, food with raw

¹Canada, Health. "Safe Cooking Temperatures." *Canada.ca*, / Gouvernement Du Canada, 29 May 2020, <https://www.canada.ca/en/health-canada/services/general-food-safety-tips/safe-internal-cooking-temperatures.html>.

eggs in them has to be cooked to at least 74°C , if not higher. This information gave me some idea about what temperature my cookies should reach. With further research, I found a blog named ThermoBlog, belonging to the thermometer company, ThermoWorks. An employee at ThermoWorks, Martin Earl, wrote an entry deducing that as soon as the internal temperature of cookies are within a range of 79°C to 85°C , the eggs in the cookie batter coagulate and the starches gelatinize², indicating that the raw ingredients in the batter are now safe to consume. Once the chocolate chip cookies are within the range of the ideal internal temperature, I can be assured that the raw ingredients are baked thoroughly without having to leave my cookies in the oven for longer, which increases its likelihood of getting burnt. The aim of my exploration is to model the internal temperature of the cookies over time and to calculate the precise amount of time needed for the cookies to be baked to perfection.

Collection of Data:

Since the aim of my exploration is to discover the amount of time it takes for cookies to reach internal temperatures of 79°C to 85°C within the typical circumstances I bake them in, it's necessary for me to collect primary data by conducting an experiment. In this way, I can be sure that the answer to my aim, the optimal amount of time for baking cookies, applies to my usual baking routine. The experiment consisted of carefully preparing the chocolate chip cookie dough according to my favourite recipe before placing them into an oven heated to 325°F or 162.778°C . I would then

² Earl, Martin. "Chocolate Chip Cookies: Heat Is An Ingredient." *ThermoWorks*, <https://blog.thermoworks.com/desserts/chocolate-chip-cookies/>

measure their internal temperatures with a digital cooking thermometer that has a measurement uncertainty of $\pm 1^\circ\text{C}$. The recipe had suggested fifteen minutes of baking time for its yield of twelve cookies, so I measured and recorded each cookie's temperature fifteen times, one for each minute interval. I used specific measurements to make sure that each ball of dough for the twelve cookies were the same size in an attempt to reduce systematic errors; each cookie was made with exactly one-quarter cup of dough. As I was designing my experiment, I initially planned on placing all twelve cookies into the oven at once, opening the oven door every minute to stick the cooking thermometer into each cookie and record its temperature. However, this would tarnish the accuracy and quality of the results because opening the oven door every minute will only allow heat to escape and cold air to enter, thus creating a misrepresentation of the cookies' internal temperature and likely prolonging the actual baking time of the cookies. Additionally, taking the temperature of all twelve cookies would take more than a minute, leaving no time for the cookies to bake in a closed oven.

I revised my experiment by baking the cookies one at a time and inserting the probe of the cooking thermometer into the center of each mound of dough for the entire duration of its baking time without removing it. Repeating the experiment twelve times, even when under the same conditions, will produce internal temperatures that vary for each cookie due to potential experimental errors I'm unable to restrict. Once I collect all the data, I can calculate and graph the mean temperatures of all twelve cookies at each minute in order to arrive at the best approximation of the internal temperature. Because the cooking thermometer I used had a long insulated wire

attaching the metal probe at one end and the display screen showing the temperature on the other end, the oven door could be closed while the probe is inserted in the cookie in the oven and the display screen is outside of the oven where I can read the temperature. By sticking the probe into the very center of the cookie, it ensures that the thermometer is taking the temperature of the actual cookie and not the temperature of the oven or baking tray. As soon as I placed a cookie into the oven, I began the stopwatch on my phone and for every minute that passed, I recorded the temperature shown on the display screen. I decided against using a timer because resetting the timer could possibly waste a few seconds, causing inexact intervals of a minute. The raw internal temperatures for each of the twelve cookies over fifteen minutes are recorded in the tables located in the appendices.

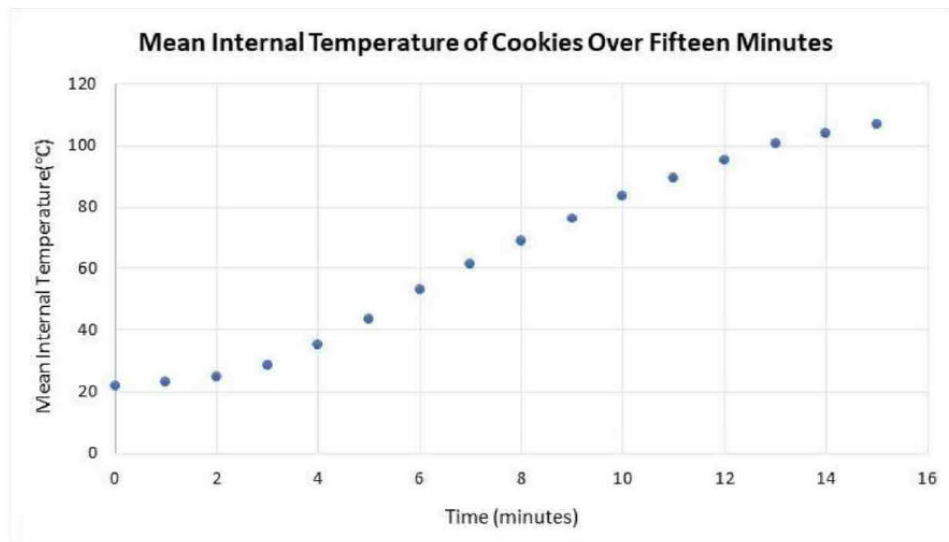
Results:

Before being placed into the oven, the temperature of each ball of cookie dough was measured to be at the same temperature as the room it was prepared in, 22°C . The mean internal temperature at each minute is rounded to two decimal places. It is unrealistic that temperature is recorded only in whole numbers because temperature doesn't increase or decrease by exactly 1°C every time; it changes gradually and decimal values can help to show that gradual change. I thought that two decimal places would retain an appropriate amount of accuracy.

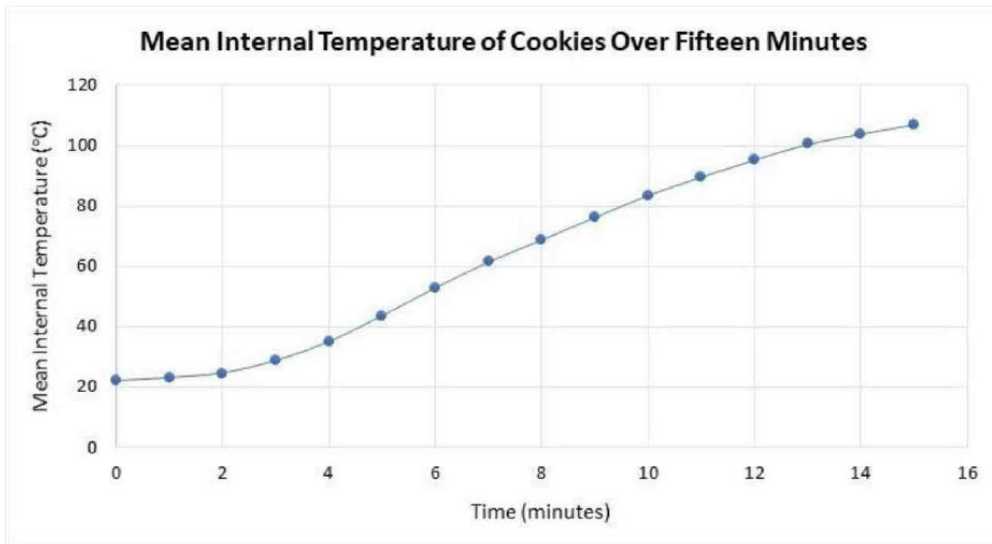
Mean Internal Temperature of Cookies Over Fifteen Minutes

Time (minutes)	Mean Internal Temperature ($^{\circ}\text{C}$)	Time (minutes)	Mean Internal Temperature ($^{\circ}\text{C}$)
0	22.00	8	68.75
1	23.00	9	76.25
2	24.58	10	83.50
3	28.67	11	89.67
4	35.00	12	95.33
5	43.58	13	100.50
6	52.83	14	103.92
7	61.42	15	107.08

In the table above, the mean internal temperatures of all twelve cookies at every minute has been calculated and recorded. Next, the data was graphed in a discrete scatter graph.



After examining the curve in the graph above, it appears to be a sinusoidal or a cosine function's graph. To be certain, I graphed the same data points but this time, a continuous line connecting all of them was added to clearly see the curve of the data.



Modeling:

Model One: Sine Curve

In the first attempt to find a function that models the data, I will be considering a sine model. The general function for a sine curve is: $f(x) = a \sin [b(x - c)] + d$.

To find the amplitude (a) or the vertical stretch factor for the function, the range of the data has to be divided by 2. The range can be found by subtracting the y -coordinate of the minimum point (0,22) from the y -coordinate of the maximum point (15, 107.08). The range is 85.08 (107.08–22).

It is then divided by 2 (85.08 ÷ 2) and is calculated to be 42.54.

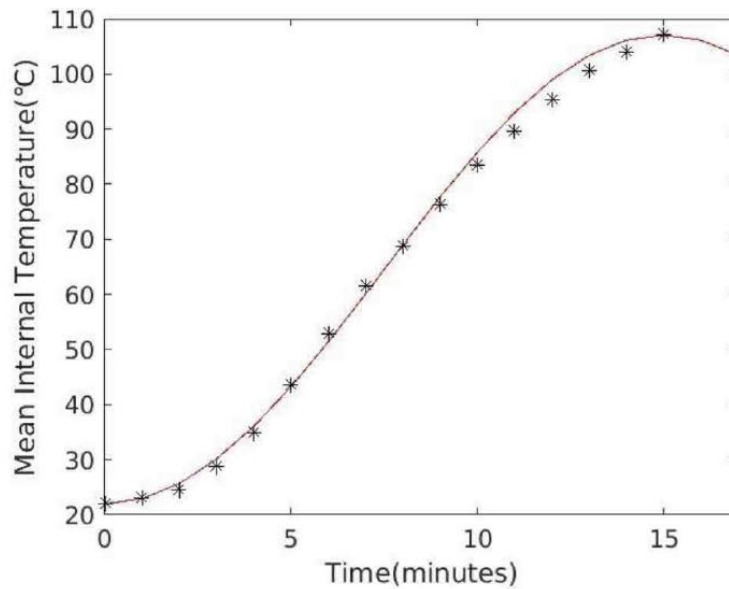
Next, I have to calculate the value of the horizontal stretch which is represented by the coefficient b .

It can be found by using the equation $b = \frac{2\pi}{\text{period}}$. Assuming that the first and last data point in the graph above are the minimum and maximum points, respectively, the graph only displays half of the period of a sine curve. I have to multiply the value of half of the period by 2. Half of the period is equal to 15 and so, a full period will be equal to 30 (15×2). Substituting the value of the period into the equation $b = \frac{2\pi}{\text{period}}$ will give me $b = \frac{2\pi}{30}$ which can be reduced to $b = \frac{\pi}{15}$.

Because the minimum point of the graph occurs at $x = 0$ instead of at the usual x value of $-\frac{\pi}{2}$ in the graph of $\sin(x)$, the horizontal phase shift, represented by b , is $\frac{\pi}{2}$ to the right. The value of the horizontal phase shift is $\frac{1}{4}$ of the period to the right because $\frac{\pi}{2} \div 2\pi = \frac{1}{4}$; $\frac{1}{4}$ of the period, 30, is 7.5, so $b = 7.5$.

The constant d represents the vertical translation of the graph. In order to find d , I have to take into account the horizontal phase shift that caused a minimum point to be on the y -axis. At this point, the minimum point is at a y -value of -42.54 (due to the amplitude) so the graph has to be translated upwards by 64.54 ($42.54 + 22$) for the minimum point to reach its y -value of 22 ; thus, $d = 64.54$.

The complete function is $f(x) = 42.54 \sin\left[\frac{\pi}{15}(x - 7.5)\right] + 64.54$. The graph comparing the function and the actual data points is shown below; the function is presented in red and the data points are displayed with asterisks.



As seen above in the graph, the sine function I determined fits well with the lower half of the data points ($x=1$ to $x=9$) but the upper half of the curve ($x=10$ to $x=15$) is slightly higher than the actual data points. Furthermore, the sine curve may model the data points within the domain of $[0,15]$ but once the domain extends past $x=15$, the curve is no longer an accurate model of the mean internal temperature over time. The cookies were baked in an oven set to a temperature of 325°F or 162.778°C , so logically, the temperature of the cookies could rise until 162.778°C before levelling off if I left them in the oven for longer. If I was modelling the data beyond $x=15$, I would have to find a different function to account for the increase in temperature until 162.778°C since the sine curve decreases after $x=15$. However, I'm only focusing on the domain of $[0,15]$, therefore, I consider the sine function as a decent model for the data.

Model Two: Cubic Function

To improve the modeling of data, the function I decided to try next is a cubic function. The standard cubic function is $f(x) = ax^3 + bx^2 + cx + d$. From the data, it's known that the y-intercept of a graph that would model the data has to be $y=22$ since the internal temperature at 0 minutes is 22°C . The y-intercept is represented by the constant d and so, $d = 22$. The y-intercept $(0,22)$ is also considered the minimum point of the graph while the last data point $(15,107.08)$ is the maximum point. At the maximum and minimum points, the first derivative of the graph's function is equal to 0. Since I don't know the graph's function yet, I will work backwards by first figuring out the first derivative using the maximum and minimum values before determining its indefinite integral, which will lead me to the function of the graph. Because the x-values of the maximum and minimum points are 0 and 15, the critical numbers are also 0 and 15, meaning that the first derivative in factored form is $f'(x) = h(x-0)(x-15)$ or $f'(x) = h(x)(x-15)$, where h is a constant factor. Because h is a constant, I will leave it as a factor while I expand $(x)(x-15)$ to $x^2 - 15x$. I can now determine the indefinite integral of $x^2 - 15x$.

$$\begin{aligned} & h \int x^2 - 15x \, dx \\ &= h \left(\frac{1}{3}x^3 - \frac{15}{2}x^2 + c \right) \\ f(x) &= h \left(\frac{1}{3}x^3 - \frac{15}{2}x^2 + c \right) \end{aligned}$$

From here, I will substitute the x and y values of the minimum point $(0,22)$ into the function.

$$22 = h\left(\frac{1}{3}(0)^3 - \frac{15}{2}(0)^2 + c\right)$$

$$22 = hc$$

There are two unknown variables in the equation, making it difficult to solve, so I will eliminate one of the variables by making $c = \frac{22}{h}$. Then, to find h , I will substitute the x and y values of the maximum point $(15, 107.08)$ into the function along with $\frac{22}{h}$ in substitution for c .

$$107.08 = h\left(\frac{1}{3}(15)^3 - \frac{15}{2}(15)^2 + \frac{22}{h}\right)$$

$$107.08 = h(-562.5 + \frac{22}{h})$$

$$107.08 = -562.5h + 22$$

$$85.08 = -562.5h$$

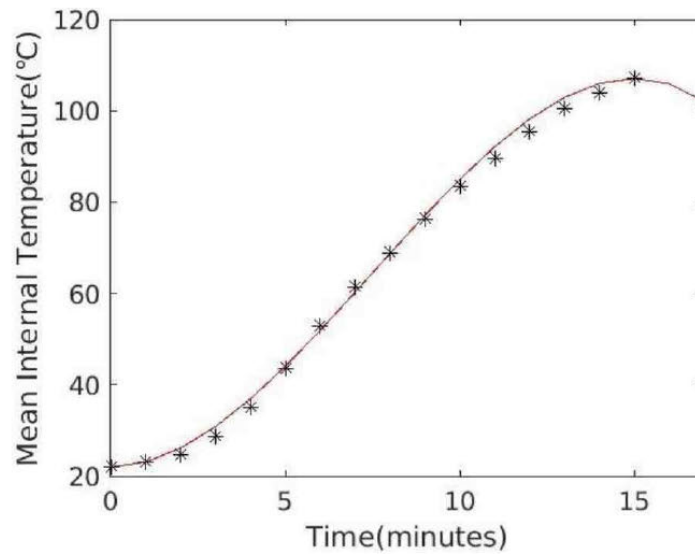
$$-\frac{85.08}{562.5} = h$$

Now, I can calculate the value of c using h .

$$c = 22 \div -\frac{85.08}{562.5}$$

$$c = -\frac{12375}{85.08}$$

The cubic function is $f(x) = -\frac{85.08}{562.5}\left(\frac{1}{3}x^3 - \frac{15}{2}x^2 - \frac{12375}{85.08}\right)$. All the values are unrounded in order to produce the most accurate model. Once again, I graphed the function with the data from the experiment.



At first glance, the cubic function seems identical to the sine function but with further inspection, I observed that the cubic curve actually touches the data points on the upper half unlike the sine curve. It doesn't correspond with each point perfectly but it is still an improvement from the sine function as it is closer to the data points. Similar to what I explained in the sine model, the cubic function is unable to model accurately if the domain is broadened past $x=15$. Once past $x=15$, the y -values of the cubic function, representing the temperature, begin to decrease instead of increasing until 162.778°C ; however, it's an excellent model for the data in the domain of $[0,15]$.

Model Three: Logistic Function:

While using the calculus textbook³ to do my work, I came across a section that explained the logistic function. I was intrigued as this equation had not been taught in class and decided to read

³ "Chapter 9.6." *Calculus: a First Course*, by James Stewart et al., McGraw-Hill Ryerson, 1989, pp.426–429, <https://sites.google.com/a/share.epsb.ca/wilde-math/my-subjects/math-31-ap-ab/calculus---a-first-course-textbook>.

more about it. A picture of the curve the equation produced, which I later discovered is called a sigmoid curve, was displayed and it appeared to be very similar to the curve of my data. Although the logistic function is mainly used to model population growth, I decided to try using it to model my data and see if it would be a better fit. Since it models population growth, the function considers the fact that an environment is only able to support a certain number of individuals and as the population advances towards this number, the growth rate decreases to zero and the graph levels off. As mentioned before, the internal temperature of the cookies could potentially reach 162.778°C before the graph evens off at a horizontal asymptote. Thus, I thought that the logistic function would accurately depict the relationship between internal temperature and time to an extent. The general logistic function is $f(x) = \frac{K}{1+Ce^{-qx}}$. To begin finding the function, I must determine the maximum temperature the cookies can rise to, represented by the variable K . K is equal to 162.778 since the temperature of the cookies can't surpass the temperature of its environment, the oven, which is set to 162.778°C . Then, I can substitute the initial temperature $(0, 22)$ in to find C .

$$22 = \frac{162.778}{1+Ce^{-q(0)}}$$

$$22 = \frac{162.778}{1+Ce^0}$$

$$22(1+C) = 162.778$$

$$1+C = \frac{162.778}{22}$$

$$C = \frac{140.778}{22}$$

$$C = 6.399$$

To determine q , I will substitute in another data point (10, 83.5).

$$83.5 = \frac{162.778}{1+6.399e^{-q(10)}}$$

$$83.5(1 + 6.399e^{-10q}) = 162.778$$

$$1 + 6.399e^{-10q} = \frac{162.778}{83.5}$$

$$6.399e^{-10q} = \frac{79.278}{83.5}$$

$$e^{-10q} = \frac{79.278}{534.3165}$$

$$\ln(e^{-10q}) = \ln\left(\frac{79.278}{534.3165}\right)$$

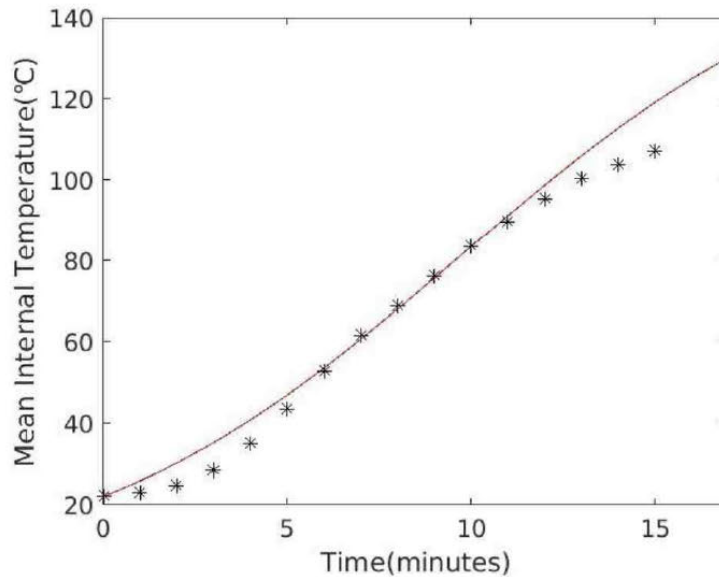
$$-10q \cdot \ln e = \ln\left(\frac{79.278}{534.3165}\right)$$

$$q = \frac{\ln(79.278) - \ln(534.3165)}{-10}$$

$$q = 0.1908027697$$

Altogether, the complete logistic function is $f(x) = \frac{162.778}{1+6.399e^{-0.1908027697x}}$. The values are unrounded

again to maintain precision. Below is the graph showing the function and the data points.



It's apparent from the graph above that aside from the six data points in the middle, the function doesn't match the rest of the data satisfactorily. The relationship between the two variables in a logistic function may still be the true relationship between the internal temperature of cookies and time but, it is seen through this attempt at modelling that the logistic function doesn't correspond well with the specific data I've collected. This could be because while calculating the parameters of the function, I may have substituted a flawed data point into the function. The experiment I conducted would have some experimental errors out of my control such as the oven temperature fluctuating instead of staying constant, causing some of the data to be incorrect. I had tried to reduce the error by taking the mean of all the results and yet, a few outlying results is enough to skew up the mean. So, the data point I chose to substitute into the function could have been an outlier, thus it would produce an inaccurate function that would misrepresent the model.

Analysis and Conclusion:

To decide on which model most accurately depicts the data, a table was created below to compare the recorded internal temperature of the cookies with the values of each of the three functions.

Time (mins)	Recorded Internal Temp. ($^{\circ}\text{C}$)	Sine Model Internal Temp. ($^{\circ}\text{C}$)	Cubic Model Internal Temp. ($^{\circ}\text{C}$)	Logistic Model Temp. ($^{\circ}\text{C}$)
0	22	22	22	22
1	23	22.93	23.084	25.889

2	24.58	25.678	26.134	30.318
3	28.67	30.124	30.848	35.309
4	35	36.075	36.924	40.868
5	43.58	43.27	44.058	46.98
6	52.83	51.394	51.948	53.604
7	61.42	60.093	60.292	60.672
8	68.75	68.987	68.788	68.091
9	76.25	77.686	77.132	75.745
10	83.5	85.81	85.022	83.5
11	89.67	93.005	92.156	91.217
12	95.33	98.956	98.232	98.759
13	100.5	103.402	102.946	106
14	103.92	106.15	105.996	112.837
15	107.08	107.08	107.08	119.189

I calculated the difference between the values of each function and the recorded temperatures before taking the average of all the differences. The tables displaying the exact differences between the values of all three functions and the recorded temperatures are located in the appendices.

	Sine Model	Cubic Model	Logistic Model
Mean Difference of Values/Error	-1.035°C	-1.035°C	-3.431°C

From the mean difference of values or mean error, it's clear that the logistic model is the least suitable model for the data. Its mean error is higher than both the sine and cubic model's, indicating that the values of the logistic function have more discrepancy with or are further away from the recorded temperatures. A possible reason for why the logistic model didn't match well could be because logistic functions are mainly used to model population growth but I had used it to model the rise of the internal temperature of cookies. I had thought that the curve of a logistic function seemed similar to the curve of my data and that a logistic function would reveal the relationship between internal temperature and time; however, population growth and rising temperatures are two different topics and likely have different relationships. As explained before, the data I collected is not perfect as errors I'm incapable of restricting during the experiment could have distorted the results. By substituting in random data points to find the function, imprecise data points could ruin the model of the function. Contrary to my previous thoughts about the cubic model being a better fit for the data, the sine and cubic model have the same mean error. The sine model had a lower mean error than the logistic model because the sine function included more parameters, which could help stretch and move the function to fit the graph better. The sine model also didn't require any data points to be substituted in, risking a misrepresentation due to skewed data. Like the sine function, the cubic function had a few more parameters that helped it to match the data curve, therefore, its mean error was low. However, the cubic function required substitution of the maximum and minimum points, which brought up the risk of faulty data, but nevertheless, the function modeled the data nicely. Because both the sine and cubic model had the same mean error, I

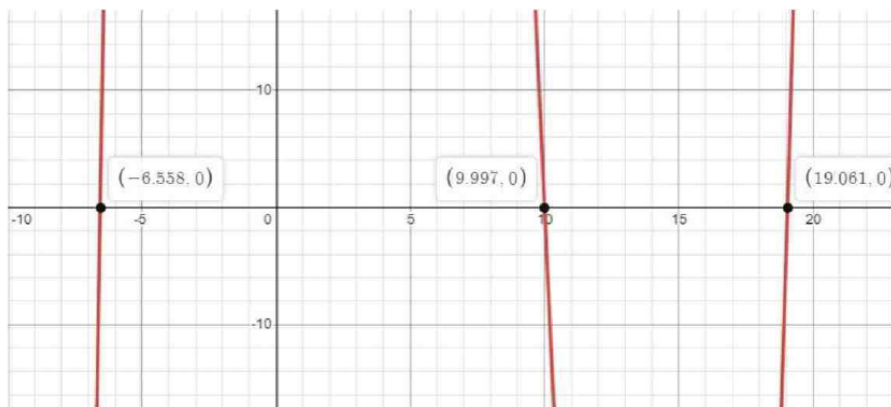
will have to use my own judgment to decide on which function I use to model the data. Examining the sine and cubic curves once again, I have resolved that the cubic function is the most suitable model and equation to accomplish the aim I set in the beginning of this exploration: find the optimal time for baking cookies. I made this decision because I think that the values of the cubic function are closest to the data points within the section ($y=79$ to $y=85$) of the graph I would get the answer to my aim from. For example, the recorded temperature had been 83.5° and the value of the cubic function was 85.022 while the value of the sine function was 85.81 . In my introduction, my research showed that when the internal temperature of cookies reaches between 79°C and 85°C , the cookies are baked thoroughly and are safe to eat. For the purposes of this exploration, I will establish 85°C as the desired internal temperature my cookies should reach.

$$85 = -\frac{85.08}{562.5} \left(\frac{1}{3}x^3 - \frac{15}{2}x^2 - \frac{12375}{85.08} \right)$$

$$-\frac{47812.5}{85.08} = \frac{1}{3}x^3 - \frac{15}{2}x^2 - \frac{12375}{85.08}$$

$$0 = \frac{1}{3}x^3 - \frac{15}{2}x^2 + \frac{35437.5}{85.08}$$

To find the x-values, I graphed $T(x) = \frac{1}{3}x^3 - \frac{15}{2}x^2 + \frac{35437.5}{85.08}$ and found the zeros or x-intercepts of the graph.



Shown in the graph above, the x-intercepts are approximately $(-6.558, 0)$, $(9.997, 0)$, and $(19.061, 0)$. Within the context of my problem, the x-intercept $(-6.558, 0)$ cannot be applied because the x-axis of the cubic function represents time and it's impossible for time to be a negative value. The x-intercept $(19.061, 0)$ is not applicable to the context as well since it's located outside of the domain I had previously established, $[0, 15]$. Therefore, the x-value or time when the cookies would reach an internal temperature of 85°C is at 9.997 minutes or if rounded to three significant digits, 10.0 minutes.

Evaluation (Limitations and Extensions):

Throughout the experiment, there were several factors I was unable to regulate. The temperature of the oven may not have stayed constant and changes in the oven temperature can affect the amount of time it takes for the cookies to heat up. Moreover, because I baked each cookie one by one, the last cookie I placed into the oven would have increased in temperature faster due to a warmer oven compared to the first cookie in a cooler oven. The sample size in my experiment was relatively small with only twelve cookies; if more cookies had been made, expanding the sample size, I would have collected more reliable data, which would result in an improved data model. The cooking thermometer I used has its own limitations with a measurement uncertainty of $\pm 1^{\circ}\text{C}$, implying that the temperature the thermometer measures has a chance of it being 1°C higher or lower than the actual temperature. This value of uncertainty is rather large and it decreases the credibility and exactness of the measured temperatures. As I was researching the sigmoid curve, I

discovered that there were other types of functions that could produce the same curve such as the arctangent function. I didn't explore those functions as they seemed more advanced but in the future, when my math knowledge expands, I could attempt to model the same data with these new functions and see if they would be a better fit. Another possible extension could be baking a different type of cookie and modeling its internal temperature over time before comparing it to the model of the chocolate chip cookies. I would be able to examine the differences and similarities, as well as explore the reasons behind these differences such as different ingredients needing different amounts of time to be cooked.

As I worked on this exploration, I was amazed at how so much math, such as integrals and Euler's number, could be hidden behind a simple activity like baking. I am grateful for the opportunity to explore math by myself in a unique way and it has taught me to be observant of the possibilities of math applications in everyday objects or activities around me.

Appendices:

Cookie 1:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	24	25	27	32	42	52	61	69	77	86	92	96	99	102

Cookie 2:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	23	24	26	28	32	39	48	56	65	74	82	87	95	96

Cookie 3:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	24	25	29	35	40	45	51	57	63	69	76	82	88	93

Cookie 4:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	24	25	27	30	34	39	43	49	54	60	67	73	78	83	88

Cookie 5:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	24	24	25	28	32	40	52	66	81	88	94	98	102	105	110

Cookie 6:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	24	24	26	33	43	54	64	73	83	94	101	104	107	108	114

Cookie 7:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	33	55	75	81	84	87	90	97	104	108	110	112	113	118

Cookie 8:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	22	23	25	30	45	63	77	82	88	94	98	105	110	112	114

Cookie 9:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	26	36	52	72	79	83	85	90	96	100	104	109	111	113

Cookie 10:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	22	22	24	26	33	44	55	66	75	81	88	98	105	109	110

Cookie 11:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	22	23	25	31	44	59	71	77	81	88	93	99	107	110	112

Cookie 12:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	24	27	33	44	58	69	77	84	92	98	103	111	114	115

Comparison of the Sine Model Values with the Recorded Temperatures:

Time (mins)	Recorded Internal Temperature (°C)	Internal Temperature from Model (°C)	Difference of Values (°C)
0	22	22	0
1	23	22.93	0.07
2	24.58	25.678	-1.098
3	28.67	30.124	-1.454
4	35	36.075	-1.075
5	43.58	43.27	0.31
6	52.83	51.394	1.436
7	61.42	60.093	1.327
8	68.75	68.987	-0.237

9	76.25	77.686	-1.436
10	83.5	85.81	-2.31
11	89.67	93.005	-3.335
12	95.33	98.956	-3.626
13	100.5	103.402	-2.902
14	103.92	106.15	-2.23
15	107.08	107.08	0

Comparison of the Cubic Model Values with the Recorded Temperatures:

Time (mins)	Recorded Internal Temperature ($^{\circ}\text{C}$)	Internal Temperature from Model ($^{\circ}\text{C}$)	Difference of Values ($^{\circ}\text{C}$)
0	22	22	0
1	23	23.084	-0.084
2	24.58	26.134	-1.554
3	28.67	30.848	-2.178
4	35	36.924	-1.924
5	43.58	44.058	-0.478
6	52.83	51.948	0.882
7	61.42	60.292	1.128
8	68.75	68.788	-0.038
9	76.25	77.132	-0.882
10	83.5	85.022	-1.522

11	89.67	92.156	-2.486
12	95.33	98.232	-2.902
13	100.5	102.946	-2.446
14	103.92	105.996	-2.076
15	107.08	107.08	0

Comparison of the Logistic Model Values with the Recorded Temperatures:

Time (mins)	Recorded Internal Temperature (°C)	Internal Temperature from Model (°C)	Difference of Values (°C)
0	22	22	0
1	23	25.889	-2.889
2	24.58	30.318	-5.738
3	28.67	35.309	-6.639
4	35	40.868	-5.868
5	43.58	46.98	-3.4
6	52.83	53.604	-0.774
7	61.42	60.672	0.748
8	68.75	68.091	0.659
9	76.25	75.745	0.505
10	83.5	83.5	0
11	89.67	91.217	-1.547
12	95.33	98.759	-3.429

13	100.5	106	-5.5
14	103.92	112.837	-8.917
15	107.08	119.189	-12.109

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