Modeling the Internal Temperature of Cookies Over Time

Introduction:

From a young age, I've been an enthusiast for everything chocolate—chocolate ice cream, brownies, and of course, the classic chocolate chip cookies. Rather than purchasing the chocolate chip cookies at the grocery store, I prefer to bake them myself. The cookies are not only fresher and more delicious, but I value the time I spend baking as I find it to be a very relaxing and gratifying activity. However, despite putting my cookies in the oven for the amount of time the recipe suggests, they usually turn out burnt on the bottom, causing too many baking sessions that started with eagerness to end in frustration. Therefore, I have resolved to fix the problem of burnt cookies by using math to discover the optimal amount of time chocolate chip cookies should be baked for to ensure that they're not undercooked or burnt, but perfect.

While researching how to ensure my cookies don't burn, I wondered if there was another quantifiable way to signify when a cookie was done baking, aside from trusting the suggested time on the recipe. I thought of temperature since it's easy to measure and relevant to baking; this sparked the idea of measuring the internal temperature of the cookies until it reaches a specific temperature whereby all raw ingredients in the dough are cooked and safe to eat—but what temperature is that? Because almost all cookie dough consists of raw eggs, I have to be certain that the raw eggs are baked to a safe temperature since raw eggs can be harmful to the body due to bacteria within them that can cause food poisoning. According to Health Canada[1], food with raw

[1]Canada, Health. "Safe Cooking Temperatures ." *Canada.ca*, / Gouvernement Du Canada, 29 May 2020, https://www.canada.ca/en/health-canada/services/general-food-safety-tips/safe-interna l-cooking-temperatures.html.

eggs in them has to be cooked to at least 74℃, if not higher. This information gave me some idea about what temperature my cookies should reach. With further research, I found a blog named ThermoBlog, belonging to the thermometer company, ThermoWorks. An employee at ThermoWorks, Martin Earl, wrote an entry deducing that as soon as the internal temperature of cookies are within a range of 79℃ to 85℃, the eggs in the cookie batter coagulate and the starches gelatinize[2], indicating that the raw ingredients in the batter are now safe to consume. Once the chocolate chip cookies are within the range of the ideal internal temperature, I can be assured that the raw ingredients are baked thoroughly without having to leave my cookies in the oven for longer, which increases its likelihood of getting burnt. The aim of my exploration is to model the internal temperature of the cookies over time and to calculate the precise amount of time needed for the cookies to be baked to perfection.

Collection of Data:

Since the aim of my exploration is to discover the amount of time it takes for cookies to reach internal temperatures of 79℃ to 85℃ within the typical circumstances I bake them in, it's necessary for me to collect primary data by conducting an experiment. In this way, I can be sure that the answer to my aim, the optimal amount of time for baking cookies, applies to my usual baking routine. The experiment consisted of carefully preparing the chocolate chip cookie dough according to my favourite recipe before placing them into an oven heated to 325℉ or 162.778℃. I would then

[2] Earl, Martin. "Chocolate Chip Cookies: Heat Is An Ingredient." *ThermoWorks*, https://blog.thermoworks.com/desserts/chocolate-chip-cookies/

measure their internal temperatures with a digital cooking thermometer that has a measurement uncertainty of ±1°C. The recipe had suggested fifteen minutes of baking time for its yield of twelve cookies, so I measured and recorded each cookie's temperature fifteen times, one for each minute interval. I used specific measurements to make sure that each ball of dough for the twelve cookies were the same size in an attempt to reduce systematic errors; each cookie was made with exactly one-quarter cup of dough. As I was designing my experiment, I initially planned on placing all twelve cookies into the oven at once, opening the oven door every minute to stick the cooking thermometer into each cookie and record its temperature. However, this would tarnish the accuracy and quality of the results because opening the oven door every minute will only allow heat to escape and cold air to enter, thus creating a misrepresentation of the cookies' internal temperature and likely prolonging the actual baking time of the cookies. Additionally, taking the temperature of all twelve cookies would take more than a minute, leaving no time for the cookies to bake in a closed oven.

I revised my experiment by baking the cookies one at a time and inserting the probe of the cooking thermometer into the center of each mound of dough for the entire duration of its baking time without removing it. Repeating the experiment twelve times, even when under the same conditions, will produce internal temperatures that vary for each cookie due to potential experimental errors I'm unable to restrict. Once I collect all the data, I can calculate and graph the mean temperatures of all twelve cookies at each minute in order to arrive at the best approximation of the internal temperature. Because the cooking thermometer I used had a long insulated wire attaching the metal probe at one end and the display screen showing the temperature on the other end, the oven door could be closed while the probe is inserted in the cookie in the oven and the display screen is outside of the oven where I can read the temperature. By sticking the probe into the very center of the cookie, it ensures that the thermometer is taking the temperature of the actual cookie and not the temperature of the oven or baking tray. As soon as I placed a cookie into the oven, I began the stopwatch on my phone and for every minute that passed, I recorded the temperature shown on the display screen. I decided against using a timer because resetting the timer could possibly waste a few seconds, causing inexact intervals of a minute. The raw internal temperatures for each of the twelve cookies over fifteen minutes are recorded in the tables located in the appendices.

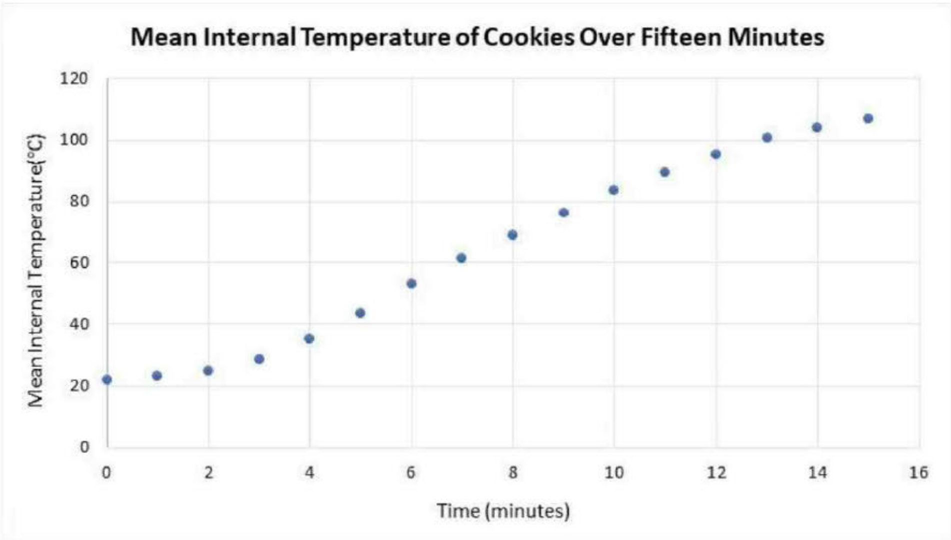
Results:

Before being placed into the oven, the temperature of each ball of cookie dough was measured to be at the same temperature as the room it was prepared in, 22℃. The mean internal temperature at each minute is rounded to two decimal places. It is unrealistic that temperature is recorded only in whole numbers because temperature doesn't increase or decrease by exactly 1℃ every time; it changes gradually and decimal values can help to show that gradual change. I thought that two decimal places would retain an appropriate amount of accuracy.

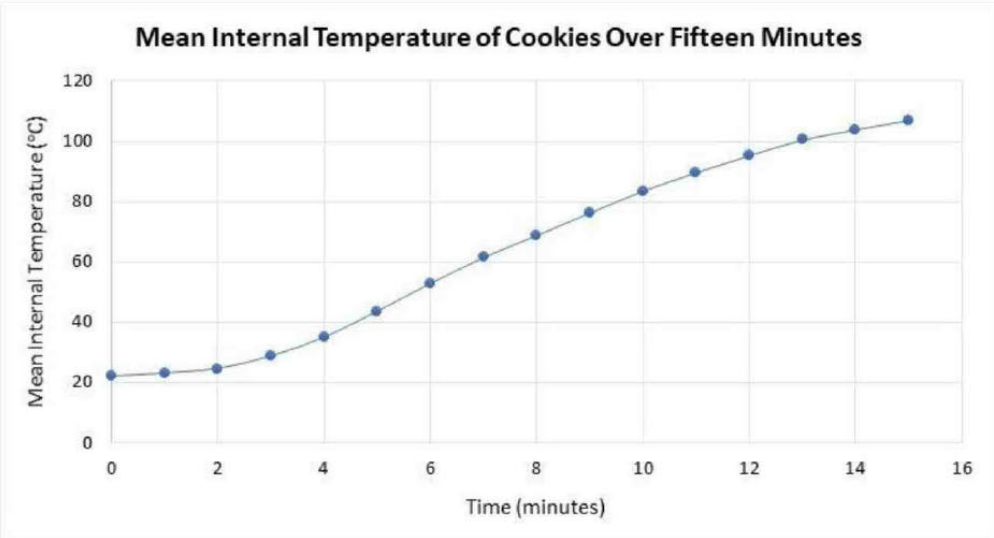
Mean Internal Temperature of Cookies Over Fifteen Minutes

Time (minutes)	Mean Internal Temperature (℃)	Time (minutes)	Mean Internal Temperature (℃)
0	22.00	8	68.75
1	23.00	9	76.25
2	24.58	10	83.50
3	28.67	11	89.67
4	35.00	12	95.33
5	43.58	13	100.50
6	52.83	14	103.92
7	61.42	15	107.08

In the table above, the mean internal temperatures of all twelve cookies at every minute has been calculated and recorded. Next, the data was graphed in a discrete scatter graph.



After examining the curve in the graph above, it appears to be a sinusoidal or a cosine function's graph. To be certain, I graphed the same data points but this time, a continuous line connecting all of them was added to clearly see the curve of the data.



Modeling:

Model One: Sine Curve

In the first attempt to find a function that models the data, I will be considering a sine model. The general function for a sine curve is: $f(x) = a\sin[b(x-c)]+d$.

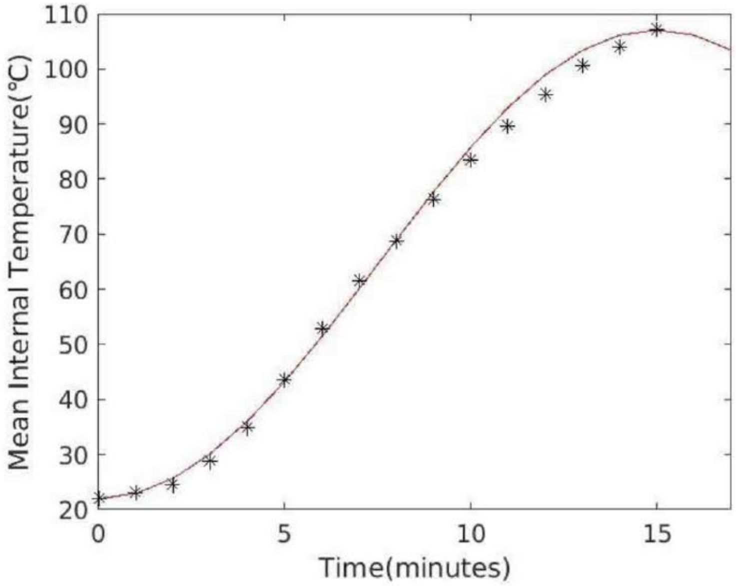
To find the amplitude (a) or the vertical stretch factor for the function, the range of the data has to be divided by 2. The range can be found by subtracting the y-coordinate of the minimum point (0,22) from the y-coordinate of the maximum point (15, 107.08). The range is 85.08 ($107.08-22$).

It is then divided by 2 ($85.08 \div 2$) and is calculated to be 42.54.

Next, I have to calculate the value of the horizontal stretch which is represented by the coefficient b. It can be found by using the equation $b = \frac{2\pi}{period}$. Assuming that the first and last data point in the graph above are the minimum and maximum points, respectively, the graph only displays half of the period of a sine curve. I have to multiply the value of half of the period by 2. Half of the period is equal to 15 and so, a full period will be equal to 30 (15×2). Substituting the value of the period into the equation $b = \frac{2\pi}{period}$ will give me $b = \frac{2\pi}{30}$ which can be reduced to $b = \frac{\pi}{15}$.

Because the minimum point of the graph occurs at $x = 0$ instead of at the usual x value of $-\frac{\pi}{2}$ in the graph of $\sin(x)$, the horizontal phase shift, represented by b, is $\frac{\pi}{2}$ to the right. The value of the horizontal phase shift is $\frac{1}{4}$ of the period to the right because $\frac{\pi}{2} \div 2\pi = \frac{1}{4}$; $\frac{1}{4}$ of the period, 30, is 7.5, so b =7.5.

The constant d represents the vertical translation of the graph. In order to find d, I have to take into account the horizontal phase shift that caused a minimum point to be on the y-axis. At this point, the minimum point is at a y-value of -42.54(due to the amplitude) so the graph has to be translated upwards by 64.54 $(42.54 + 22)$ for the minimum point to reach its y-value of 22; thus, $d = 64.54$. The complete function is $f(x) = 42.54 \sin\left[\frac{\pi}{15}(x - 7.5)\right] + 64.54$. The graph comparing the function and the actual data points is shown below; the function is presented in red and the data points are displayed with asterisks.



As seen above in the graph, the sine function I determined fits well with the lower half of the data points (x=1 to x=9) but the upper half of the curve (x=10 to x=15) is slightly higher than the actual data points. Furthermore, the sine curve may model the data points within the domain of [0,15] but once the domain extends past x=15, the curve is no longer an accurate model of the mean internal temperature over time. The cookies were baked in an oven set to a temperature of 325℉ or 162.778°C, so logically, the temperature of the cookies could rise until 162.778°C before levelling off if I left them in the oven for longer. If I was modelling the data beyond x=15, I would have to find a different function to account for the increase in temperature until 162.778°C since the sine curve decreases after x=15. However, I'm only focusing on the domain of [0,15], therefore, I consider the sine function as a decent model for the data.

Model Two: Cubic Function

To improve the modeling of data, the function I decided to try next is a cubic function. The standard cubic function is $f(x) = ax^3 + bx^2 + cx + d$. From the data, it's known that the y-intercept of a graph that would model the data has to be y=22 since the internal temperature at 0 minutes is 22°C. The y-intercept is represented by the constant d and so, $d = 22$. The y-intercept (0,22) is also considered the minimum point of the graph while the last data point (15,107.08) is the maximum point. At the maximum and minimum points, the first derivative of the graph's function is equal to 0. Since I don't know the graph's function yet, I will work backwards by first figuring out the first derivative using the maximum and minimum values before determining its indefinite integral, which will lead me to the function of the graph. Because the x-values of the maximum and minimum points are 0 and 15, the critical numbers are also 0 and 15, meaning that the first derivative in factored form is $f'(x) = h(x-0)(x-15)$ or $f'(x) = h(x)(x-15)$, where h is a constant factor. Because h is a constant, I will leave it as a factor while I expand $(x)(x-15)$ to $x^2 - 15x$. I can now determine the indefinite integral of $x^2 - 15x$.

$$h\int x^2 - 15x \, dx$$

$$= h(\tfrac{1}{3}x^3 - \tfrac{15}{2}x^2 + c)$$

$$f(x) = h(\tfrac{1}{3}x^3 - \tfrac{15}{2}x^2 + c)$$

From here, I will substitute the x and y values of the minimum point (0,22) into the function.

$$22 = h(\tfrac{1}{3}(0)^3 - \tfrac{15}{2}(0)^2 + c)$$

$$22 = hc$$

There are two unknown variables in the equation, making it difficult to solve, so I will eliminate one of the variables by making $c = \frac{22}{h}$. Then, to find h, I will substitute the x and y values of the maximum point (15, 107.08) into the function along with $\frac{22}{h}$ in substitution for c.

$$107.08 = h(\tfrac{1}{3}(15)^3 - \tfrac{15}{2}(15)^2 + \tfrac{22}{h})$$

$$107.08 = h(-562.5 + \tfrac{22}{h})$$

$$107.08 = -562.5h + 22$$

$$85.08 = -562.5h$$

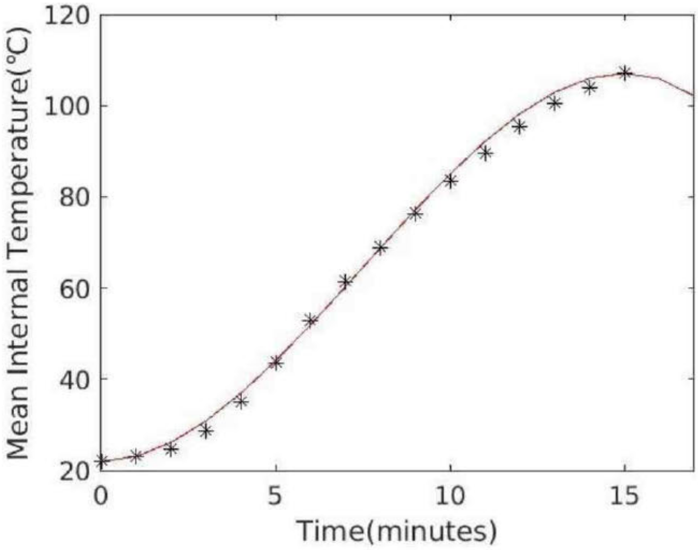
$$-\tfrac{85.08}{562.5} = h$$

Now, I can calculate the value of c using h.

$$c = 22 \div -\tfrac{85.08}{562.5}$$

$$c = -\tfrac{12375}{85.08}$$

The cubic function is $f(x) = -\frac{85.08}{562.5}(\frac{1}{3}x^3 - \frac{15}{2}x^2 - \frac{12375}{85.08})$. All the values are unrounded in order to produce the most accurate model. Once again, I graphed the function with the data from the experiment.



At first glance, the cubic function seems identical to the sine function but with further inspection, I observed that the cubic curve actually touches the data points on the upper half unlike the sine curve. It doesn't correspond with each point perfectly but it is still an improvement from the sine function as it is closer to the data points. Similar to what I explained in the sine model, the cubic function is unable to model accurately if the domain is broadened past x=15. Once past x=15, the y-values of the cubic function, representing the temperature, begin to decrease instead of increasing until 162.778°C; however, it's an excellent model for the data in the domain of [0,15].

<u>Model Three: Logistic Function:</u>

While using the calculus textbook[3] to do my work, I came across a section that explained the logistic function. I was intrigued as this equation had not been taught in class and decided to read

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[3] "Chapter 9.6." *Calculus: a First Course*, by James Stewart et al., McGraw-Hill Ryerson, 1989, pp.426–429, https://sites.google.com/a/share.epsb.ca/wilde-math/my-subjects/math-31-ap-ab/ calculus----a-first-course-textbook.

more about it. A picture of the curve the equation produced, which I later discovered is called a sigmoid curve, was displayed and it appeared to be very similar to the curve of my data. Although the logistic function is mainly used to model population growth, I decided to try using it to model my data and see if it would be a better fit. Since it models population growth, the function considers the fact that an environment is only able to support a certain number of individuals and as the population advances towards this number, the growth rate decreases to zero and the graph levels off. As mentioned before, the internal temperature of the cookies could potentially reach 162.778°C before the graph evens off at a horizontal asymptote. Thus, I thought that the logistic function would accurately depict the relationship between internal temperature and time to an extent. The general logistic function is $f(x) = \frac{K}{1+Ce^{-qx}}$ .To begin finding the function, I must determine the maximum temperature the cookies can rise to, represented by the variable K . K is equal to 162.778 since the temperature of the cookies can't surpass the temperature of its environment, the oven, which is set to 162.778°C. Then, I can substitute the initial temperature (0, 22) in to find C .

$$22 = \frac{162.778}{1+Ce^{-q(0)}}$$

$$22 = \frac{162.778}{1+Ce^{0}}$$

$$22(1 + C) = 162.778$$

$$1 + C = \frac{162.778}{22}$$

$$C = \frac{140.778}{22}$$

$$C = 6.399$$

To determine q , I will substitute in another data point (10, 83.5).

$$83.5 = \frac{162.778}{1+6.399e^{-q(10)}}$$

$$83.5(1+6.399e^{-10q}) = 162.778$$

$$1+6.399e^{-10q} = \frac{162.778}{83.5}$$

$$6.399e^{-10q} = \frac{79.278}{83.5}$$

$$e^{-10q} = \frac{79.278}{534.3165}$$

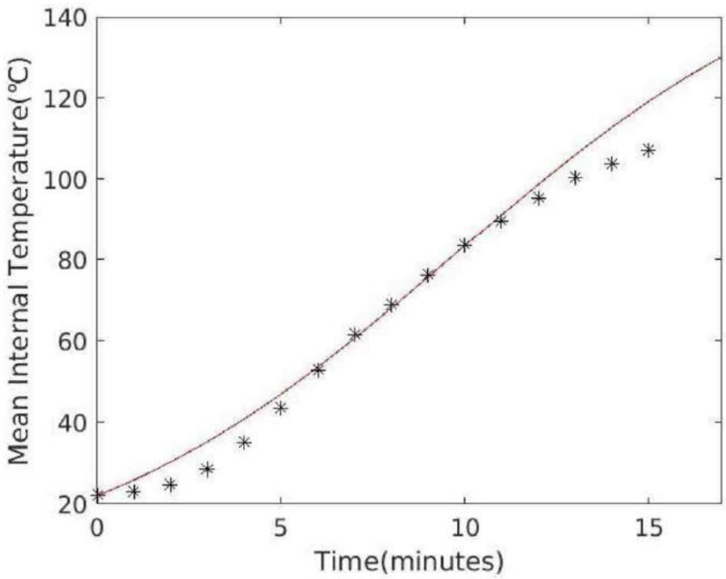
$$ln\,(e^{-10q}) = ln\,(\frac{79.278}{534.3165})$$

$$-10q \bullet ln\,e = ln\,(\frac{79.278}{534.3165})$$

$$q = \frac{ln\,(79.278) - ln\,(534.3165)}{-10}$$

$$q = 0.1908027697$$

Altogether, the complete logistic function is $f(x) = \frac{162.778}{1+6.399e^{-0.1908027697x}}$. The values are unrounded again to maintain precision.Below is the graph showing the function and the data points.



It's apparent from the graph above that aside from the six data points in the middle, the function doesn't match the rest of the data satisfactorily. The relationship between the two variables in a logistic function may still be the true relationship between the internal temperature of cookies and time but, it is seen through this attempt at modelling that the logistic function doesn't correspond well with the specific data I've collected. This could be because while calculating the parameters of the function, I may have substituted a flawed data point into the function. The experiment I conducted would have some experimental errors out of my control such as the oven temperature fluctuating instead of staying constant, causing some of the data to be incorrect. I had tried to reduce the error by taking the mean of all the results and yet, a few outlying results is enough to skew up the mean. So, the data point I chose to substitute into the function could have been an outlier, thus it would produce an inaccurate function that would misrepresent the model.

Analysis and Conclusion:

To decide on which model most accurately depicts the data, a table was created below to compare the recorded internal temperature of the cookies with the values of each of the three functions.

Time (mins)	Recorded Internal Temp. (℃)	Sine Model Internal Temp. (℃)	Cubic Model Internal Temp. (℃)	Logistic Model Temp. (℃)
0	22	22	22	22
1	23	22.93	23.084	25.889

2	24.58	25.678	26.134	30.318
3	28.67	30.124	30.848	35.309
4	35	36.075	36.924	40.868
5	43.58	43.27	44.058	46.98
6	52.83	51.394	51.948	53.604
7	61.42	60.093	60.292	60.672
8	68.75	68.987	68.788	68.091
9	76.25	77.686	77.132	75.745
10	83.5	85.81	85.022	83.5
11	89.67	93.005	92.156	91.217
12	95.33	98.956	98.232	98.759
13	100.5	103.402	102.946	106
14	103.92	106.15	105.996	112.837
15	107.08	107.08	107.08	119.189

I calculated the difference between the values of each function and the recorded temperatures before taking the average of all the differences. The tables displaying the exact differences between the values of all three functions and the recorded temperatures are located in the appendices.

	Sine Model	Cubic Model	Logistic Model
Mean Difference of Values/Error	−1.035℃	−1.035℃	−3.431℃

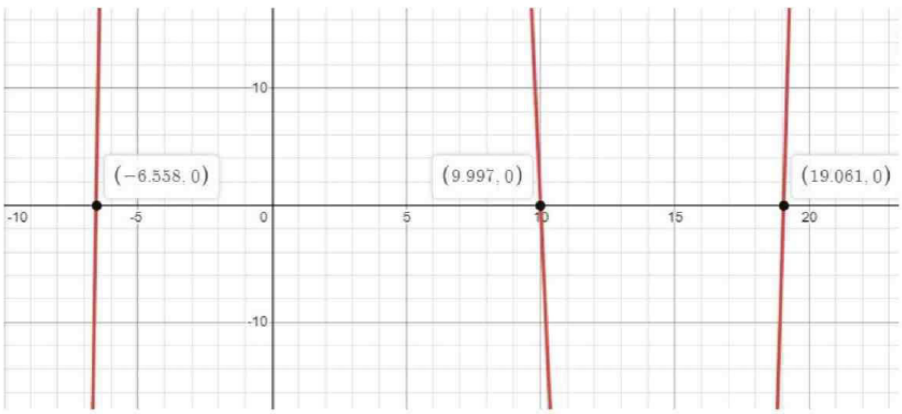
From the mean difference of values or mean error, it's clear that the logistic model is the least suitable model for the data. Its mean error is higher than both the sine and cubic model's, indicating that the values of the logistic function have more discrepancy with or are further away from the recorded temperatures. A possible reason for why the logistic model didn't match well could be because logistic functions are mainly used to model population growth but I had used it to model the rise of the internal temperature of cookies. I had thought that the curve of a logistic function seemed similar to the curve of my data and that a logistic function would reveal the relationship between internal temperature and time; however, population growth and rising temperatures are two different topics and likely have different relationships. As explained before, the data I collected is not perfect as errors I'm incapable of restricting during the experiment could have distorted the results. By substituting in random data points to find the function, imprecise data points could ruin the model of the function. Contrary to my previous thoughts about the cubic model being a better fit for the data, the sine and cubic model have the same mean error. The sine model had a lower mean error than the logistic model because the sine function included more parameters, which could help stretch and move the function to fit the graph better. The sine model also didn't require any data points to be substituted in, risking a misrepresentation due to skewed data. Like the sine function, the cubic function had a few more parameters that helped it to match the data curve, therefore, its mean error was low. However, the cubic function required substitution of the maximum and minimum points, which brought up the risk of faulty data, but nevertheless, the function modeled the data nicely. Because both the sine and cubic model had the same mean error, I will have to use my own judgment to decide on which function I use to model the data. Examining the sine and cubic curves once again, I have resolved that the cubic function is the most suitable model and equation to accomplish the aim I set in the beginning of this exploration: find the optimal time for baking cookies. I made this decision because I think that the values of the cubic function are closest to the data points within the section (y=79 to y=85) of the graph I would get the answer to my aim from. For example, the recorded temperature had been 83.5° and the value of the cubic function was 85.022 while the value of the sine function was 85.81. In my introduction, my research showed that when the internal temperature of cookies reaches between 79°C and 85°C, the cookies are baked thoroughly and are safe to eat. For the purposes of this exploration, I will establish 85°C as the desired internal temperature my cookies should reach.

$$85 = -\frac{85.08}{562.5}\left(\frac{1}{3}x^3 - \frac{15}{2}x^2 - \frac{12375}{85.08}\right)$$

$$-\frac{47812.5}{85.08} = \frac{1}{3}x^3 - \frac{15}{2}x^2 - \frac{12375}{85.08}$$

$$0 = \frac{1}{3}x^3 - \frac{15}{2}x^2 + \frac{35437.5}{85.08}$$

To find the x-values, I graphed $T(x) = \frac{1}{3}x^3 - \frac{15}{2}x^2 + \frac{35437.5}{85.08}$ and found the zeros or x-intercepts of the graph.



Shown in the graph above, the x-intercepts are approximately (−6.558, 0), (9.997, 0), and (19.061, 0). Within the context of my problem, the x-intercept (−6.558, 0) cannot be applied because the x-axis of the cubic function represents time and it's impossible for time to be a negative value. The x-intercept (19.061, 0) is not applicable to the context as well since it's located outside of the domain I had previously established, [0,15]. Therefore, the x-value or time when the cookies would reach an internal temperature of 85℃ is at 9.997 minutes or if rounded to three significant digits, 10.0 minutes.

Evaluation (Limitations and Extensions):

Throughout the experiment, there were several factors I was unable to regulate. The temperature of the oven may not have stayed constant and changes in the oven temperature can affect the amount of time it takes for the cookies to heat up. Moreover, because I baked each cookie one by one, the last cookie I placed into the oven would have increased in temperature faster due to a warmer oven compared to the first cookie in a cooler oven. The sample size in my experiment was relatively small with only twelve cookies; if more cookies had been made, expanding the sample size, I would have collected more reliable data, which would result in an improved data model. The cooking thermometer I used has its own limitations with a measurement uncertainty of ±1℃, implying that the temperature the thermometer measures has a chance of it being 1℃ higher or lower than the actual temperature. This value of uncertainty is rather large and it decreases the credibility and exactness of the measured temperatures. As I was researching the sigmoid curve, I discovered that there were other types of functions that could produce the same curve such as the arctangent function. I didn't explore those functions as they seemed more advanced but in the future, when my math knowledge expands, I could attempt to model the same data with these new functions and see if they would be a better fit. Another possible extension could be baking a different type of cookie and modeling its internal temperature over time before comparing it to the model of the chocolate chip cookies. I would be able to examine the differences and similarities, as well as explore the reasons behind these differences such as different ingredients needing different amounts of time to be cooked.

As I worked on this exploration, I was amazed at how so much math, such as integrals and Euler's number, could be hidden behind a simple activity like baking. I am grateful for the opportunity to explore math by myself in a unique way and it has taught me to be observant of the possibilities of math applications in everyday objects or activities around me.

Appendices:

Cookie 1:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	24	25	27	32	42	52	61	69	77	86	92	96	99	102

Cookie 2:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	23	24	26	28	32	39	48	56	65	74	82	87	95	96

Cookie 3:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	24	25	29	35	40	45	51	57	63	69	76	82	88	93

Cookie 4:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	24	25	27	30	34	39	43	49	54	60	67	73	78	83	88

Cookie 5:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	24	24	25	28	32	40	52	66	81	88	94	98	102	105	110

Cookie 6:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	24	24	26	33	43	54	64	73	83	94	101	104	107	108	114

Cookie 7:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	33	55	75	81	84	87	90	97	104	108	110	112	113	118

Cookie 8:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	22	23	25	30	45	63	77	82	88	94	98	105	110	112	114

Cookie 9:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	26	36	52	72	79	83	85	90	96	100	104	109	111	113

Cookie 10:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	22	22	24	26	33	44	55	66	75	81	88	98	105	109	110

Cookie 11:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	22	23	25	31	44	59	71	77	81	88	93	99	107	110	112

Cookie 12:

Time (mins)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Internal Temperature (°C)	22	23	24	27	33	44	58	69	77	84	92	98	103	111	114	115

Comparison of the Sine Model Values with the Recorded Temperatures:

Time (mins)	Recorded Internal Temperature (°C)	Internal Temperature from Model (°C)	Difference of Values (°C)
0	22	22	0
1	23	22.93	0.07
2	24.58	25.678	−1.098
3	28.67	30.124	−1.454
4	35	36.075	−1.075
5	43.58	43.27	0.31
6	52.83	51.394	1.436
7	61.42	60.093	1.327
8	68.75	68.987	−0.237

9	76.25	77.686	−1.436
10	83.5	85.81	−2.31
11	89.67	93.005	−3.335
12	95.33	98.956	−3.626
13	100.5	103.402	−2.902
14	103.92	106.15	−2.23
15	107.08	107.08	0

Comparison of the Cubic Model Values with the Recorded Temperatures:

Time (mins)	Recorded Internal Temperature (℃)	Internal Temperature from Model (℃)	Difference of Values (℃)
0	22	22	0
1	23	23.084	−0.084
2	24.58	26.134	−1.554
3	28.67	30.848	−2.178
4	35	36.924	−1.924
5	43.58	44.058	−0.478
6	52.83	51.948	0.882
7	61.42	60.292	1.128
8	68.75	68.788	−0.038
9	76.25	77.132	−0.882
10	83.5	85.022	−1.522

11	89.67	92.156	−2.486
12	95.33	98.232	−2.902
13	100.5	102.946	−2.446
14	103.92	105.996	−2.076
15	107.08	107.08	0

Comparison of the Logistic Model Values with the Recorded Temperatures:

Time (mins)	Recorded Internal Temperature (℃)	Internal Temperature from Model (℃)	Difference of Values (℃)
0	22	22	0
1	23	25.889	−2.889
2	24.58	30.318	−5.738
3	28.67	35.309	−6.639
4	35	40.868	−5.868
5	43.58	46.98	−3.4
6	52.83	53.604	−0.774
7	61.42	60.672	0.748
8	68.75	68.091	0.659
9	76.25	75.745	0.505
10	83.5	83.5	0
11	89.67	91.217	−1.547
12	95.33	98.759	−3.429

13	100.5	106	–5.5
14	103.92	112.837	–8.917
15	107.08	119.189	–12.109

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